A SIMPLE PROOF FOR MAVLO'S INEQUALITY

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MAVLO'S INEQUALITY
If $a, b > 0; n \in \mathbb{N}^*$ then:

$$\left(\frac{a+b}{2}\right)^n - (\sqrt{ab})^n \geq \frac{(\sqrt{a^n} - \sqrt{b^n})^2}{2^n}$$

Proof.
For $n = 1$:

$$\frac{a+b}{2} - \sqrt{ab} \geq \frac{(\sqrt{a} - \sqrt{b})^2}{2}$$

$$a + b - 2\sqrt{ab} \geq (\sqrt{a} - \sqrt{b})^2$$

$$(\sqrt{a} - \sqrt{b})^2 \geq (\sqrt{a} - \sqrt{b})^2$$

Suppose $n \geq 2$. Denote $a = x^2; b = y^2$.
Inequality can be written:

$$\left(\frac{x^2+y^2}{2}\right)^n - (\sqrt{x^2 \cdot y^2})^n \geq \frac{(\sqrt{x^{2n}} - \sqrt{y^{2n}})^2}{2^n}$$

$$\frac{(x^2+y^2)^n}{2^n} - x^n y^n \geq \frac{(x^n - y^n)^2}{2^n}$$

$$(x^2+y^2)^n - 2^n x^n y^n \geq x^{2n} + y^{2n} - 2x^n y^n$$

$$(x^2+y^2)^n - x^{2n} - y^{2n} \geq 2^n x^n y^n - 2x^n y^n$$

$$\sum_{k=1}^{n-1} \binom{n}{k} x^{2n-2k} y^{2k} \geq x^n y^n (2^n - 2)$$

$$2\sum_{k=1}^{n-1} \binom{n}{k} x^{2n-2k} y^{2k} \geq 2x^n y^n \cdot \sum_{k=1}^{n} \binom{n}{k}$$

$$\sum_{k=1}^{n-1} \binom{n}{k} x^{2n-2k} y^{2k} + \sum_{k=1}^{n-1} \binom{n}{k} y^{2n-2k} x^{2k} - 2\sum_{k=1}^{n-1} \binom{n}{k} x^n y^n \geq 0$$

$$\sum_{k=1}^{n-1} \binom{n}{k} x^n y^{2k} (x^{n-2k} + y^{n-2k}) + \sum_{k=1}^{n-1} \binom{n}{k} x^{2k} y^n (y^{n-2k} - x^{n-2k}) \geq 0$$

$$\sum_{k=1}^{n-1} \binom{n}{k} (x^n y^{2k} - x^{2k} y^n)(x^{n-2k} - y^{n-2k}) \geq 0$$

$$\sum_{k=1}^{n-1} \binom{n}{k} x^{2k} y^{2k} (x^{n-2k} - y^{n-2k})^2 \geq 0$$

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References

[1] Romanian Mathematical Magazine - Interactive Journal, www.ssmrmh.ro

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