

# R M M

## ROMANIAN MATHEMATICAL MAGAZINE

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If  $0 \leq a \leq \frac{\pi}{12}$  then:

$$\int_0^a \sin x \cdot \cos(6x) \cdot \cos^6(4x) \cdot \cos^{15}(2x) dx \leq \frac{1}{193} (1 - \cos^{193} a)$$

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*Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco*

Let  $x \in [0, a]$ , since  $a \leq \frac{\pi}{12} \rightarrow \cos x, \cos(2x), \cos(4x), \cos(6x), \sin x \geq 0$

$$\begin{aligned} \text{We have : } \cos(2x) &= 2 \cos^2 x - 1 \stackrel{AM-GM}{\leq} (\cos^4 x + 1) - 1 = \cos^4 x \rightarrow \cos(2x) \\ &\leq \cos^4 x \quad (1) \end{aligned}$$

$$\rightarrow \cos(4x) \stackrel{(1)}{\leq} \cos^4(2x) \stackrel{(1)}{\leq} (\cos^4 x)^4 = \cos^{16} x \rightarrow \cos(4x) \leq \cos^{16} x \quad (2)$$

$$\begin{aligned} \cos(3x) &= \cos x \cdot (4 \cos^2 x - 3) \stackrel{AM-GM}{\leq} \cos x \cdot [( \cos^8 x + 1 + 1 + 1 ) - 3] = \cos^9 x \\ &\rightarrow \cos(3x) \leq \cos^9 x \quad (i) \end{aligned}$$

$$\rightarrow \cos(6x) \stackrel{(1)}{\leq} \cos^4(3x) \stackrel{(i)}{\leq} (\cos^9 x)^4 = \cos^{36} x \rightarrow \cos(6x) \leq \cos^{36} x \quad (3)$$

$$\begin{aligned} (1), (2), (3) \rightarrow \cos(6x) \cdot \cos^6(4x) \cdot \cos^{15}(2x) &\leq \cos^{36} x \cdot (\cos^{16} x)^6 \cdot (\cos^4 x)^{15} \\ &= \cos^{192} x \end{aligned}$$

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$$\begin{aligned} \rightarrow \int_0^a \sin x \cdot \cos(6x) \cdot \cos^6(4x) \cdot \cos^{15}(2x) dx &\leq \int_0^a \sin x \cdot \cos^{192} x dx = \left[ \frac{-1}{193} \cos^{193} x \right]_0^a \\ &= \frac{1}{193} (1 - \cos^{193} a). \end{aligned}$$