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If $n \in \mathbb{N}^* - \{1\}, a \in \mathbb{R}_+, b, c, d, m, p \in \mathbb{R}_+^*, x_k \in \mathbb{R}_+^*, k = \overline{1, n},$

$X_{n,m} = \sum_{k=1}^{n} x_k^m, X_{n,p} = \sum_{k=1}^{n} x_k^p$ such that $c \cdot X_{n,p} > d \cdot \max_{1\leq k\leq n} x_k^p$ then prove:

$$\sum_{k=1}^{n} \frac{a.X_{n,m} + b.x_k{}^m}{c.X_{n,p} - d.x_k{}^p} \geq \frac{n(an+b)}{(cn-d)} \cdot \frac{X_{n,m}}{X_{n,p}}$$

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$$WLOG, \text{we may assume that}: \ x_1 \geq x_2 \geq \cdots \geq x_n \ \to \ a.X_{n,m} + b.x_1{}^m$$

$$\geq a.X_{n,m} + b.x_2{}^m \geq \cdots \geq a.X_{n,m} + b.x_n{}^m$$

$$And: \ \frac{1}{c.X_{n,p} - d.x_1{}^p} \geq \frac{1}{c.X_{n,p} - d.x_2{}^p} \geq \cdots \geq \frac{1}{c.X_{n,p} - d.x_n{}^p}$$

$$\to \text{From Chebyshev's inequality, we have}:$$

$$\sum_{k=1}^{n} \frac{a.X_{n,m} + b.x_k{}^m}{c.X_{n,p} - d.x_k{}^p} \geq \frac{1}{n}\left[\sum_{k=1}^{n}\left(a.X_{n,m} + b.x_k{}^m\right)\right]\left(\sum_{k=1}^{n}\frac{1}{c.X_{n,p} - d.x_k{}^p}\right) \overset{CBS}{\geq}$$

$$\geq \frac{1}{n}.(an+b)X_{n,m}.\frac{n^2}{\sum\left(c.X_{n,p} - d.x_k{}^p\right)} = \frac{n(an+b)X_{n,m}}{(cn-d)X_{n,p}}.$$

$$Therefore, \ \sum_{k=1}^{n} \frac{a.X_{n,m} + b.x_k{}^m}{c.X_{n,p} - d.x_k{}^p} \geq \frac{n(an+b)}{(cn-d)} \cdot \frac{X_{n,m}}{X_{n,p}}$$