PP36057

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{\cos\frac{B-C}{2}}{\sin\frac{A}{2}} = \frac{s^2+r^2-2Rr}{2Rr}$$

Solution by Daniel Sitaru.

$$\frac{\cos\frac{B-C}{2}}{\sin\frac{A}{2}} = \frac{\cos\frac{B}{2}\cos\frac{C}{2}+\sin\frac{B}{2}\sin\frac{C}{2}}{\sin\frac{A}{2}} =$$

$$= \frac{\sqrt{\frac{s(s-b)}{ac}}\cdot\sqrt{\frac{s(s-c)}{ab}}+\sqrt{\frac{(s-a)(s-c)}{ac}}\cdot\sqrt{\frac{(s-a)(s-b)}{ab}}}{\sin\frac{A}{2}} =$$

$$= \frac{\frac{s}{a}\sqrt{\frac{(s-b)(s-c)}{bc}}+\frac{s-a}{a}\sqrt{\frac{(s-b)(s-c)}{bc}}}{\sin\frac{A}{2}} =$$

$$= \frac{\frac{s}{a}\cdot\sin\frac{A}{2}+\frac{s-a}{a}\sin\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{A}{2}\cdot\left(\frac{s}{a}+\frac{s-a}{a}\right)}{\sin\frac{A}{2}} =$$

$$= \frac{b+c}{a}$$

$$\sum_{cyc}\frac{\cos\frac{B-C}{2}}{\sin\frac{A}{2}} = \sum_{cyc}\frac{b+c}{a} = \sum_{cyc}\frac{2s-a}{a} =$$

$$= 2s\sum\frac{1}{a}-3 = 2s\cdot\frac{ab+bc+ca}{abc}-3 =$$

$$= 2s\cdot\frac{s^2+r^2+4Rr}{4Rrs}-3 = \frac{s^2+r^2+4Rr}{2Rr}-3 =$$

$$= \frac{s^2+r^2-3Rr}{2Rr}$$

Equality holds for $a=b=c$. □

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