PP35630

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} a^2(2r_br_c - rr_a) = 4s^2r(R+3r)$$

Solution by Daniel Sitaru.

$$\sum_{cyc} a^2(2r_br_c - rr_a) =$$

$$= \sum_{cyc} a^2\Big(2\cdot\frac{F}{s-b}\cdot\frac{F}{s-c} - \frac{F}{s}\cdot\frac{F}{s-a}\Big) =$$

$$= F^2\sum_{cyc} a^2\Big(\frac{2}{(s-b)(s-c)} - \frac{1}{s(s-a)}\Big) =$$

$$= F^2\sum_{cyc}\frac{a^2}{F^2}(2s(s-a)-(s-b)(s-c)) =$$

$$= 2s\sum_{cyc} a^2(s-a) - \sum_{cyc} a^2(s-b)(s-c) =$$

$$= 2s\cdot 4rs(R+r) - 4rs^2(R-r) =$$

$$= 4s^2r(2R+2r-R+r) =$$

$$= 4s^2r(R+3r)$$

Equality holds for $a=b=c$.

□

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