

PP35562

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In all triangles ABC holds:

$$\cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2} < \frac{11}{4} + \frac{r}{2R}$$

Solution by Daniel Sitaru.

$$\begin{aligned} \sum_{cyc} \cos \frac{A}{2} &\leq 1 \cdot \cos \frac{A}{2} + 1 \cdot \cos \frac{B}{2} + 1 \cdot \cos \frac{C}{2} \leq \\ &\stackrel{\text{CBS}}{\leq} \sqrt{(1^2 + 1^2 + 1^2) \cdot \sum_{cyc} \cos^2 \frac{A}{2}} = \sqrt{3 \left(2 + \frac{r}{2R}\right)} < \frac{11}{4} + \frac{r}{2R} \\ &3 \left(2 + \frac{r}{2R}\right) < \left(\frac{11}{4} + \frac{r}{2R}\right)^2 \\ &6 + \frac{3r}{2R} < \frac{121}{16} + \frac{11r}{4R} + \frac{r^2}{4R^2} \\ &\frac{r^2}{4R^2} + \frac{5r}{4R} + \frac{25}{16} > 0 \\ &\frac{r^2}{R^2} + \frac{5r}{R} + \frac{25}{4} > 0 \\ &\frac{4r^2}{R^2} + \frac{20r}{R} + 25 > 0 \\ &\left(\frac{2r}{R} + 5\right)^2 > 0 \end{aligned}$$

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