PP35520

MIHÁLY BENCZE - ROMANIA

If $x \in \mathbb{R}$ then:

$$\frac{|\cos x|}{\sqrt{1+\sin^2 x}} + \frac{|\sin x|}{\sqrt{1+\cos^2 x}} \le \frac{2}{\sqrt{3}}$$

Solution by Daniel Sitaru.

$$\frac{|\cos x|}{\sqrt{1+\sin^2 x}} + \frac{|\sin x|}{\sqrt{1+\cos^2 x}} \overset{\text{CBS}}{\le}$$

$$\le \sqrt{(\cos^2 x + \sin^2 x)\left(\frac{1}{1+\sin^2 x} + \frac{1}{1+\cos^2 x}\right)} =$$

$$= \sqrt{\frac{1+\cos^2 x + 1 + \sin^2 x}{1+\cos^2 x + \sin^2 x + \sin^2 x \cos^2 x}} = \frac{\sqrt{3}}{\sqrt{2+\sin^2 x \cos^2 x}} \le \frac{2}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{\sqrt{2+\sin^2 x \cos^2 x}} \le \frac{2}{\sqrt{3}}$$

$$\frac{1}{2+\sin^2 x \cos^2 x} \le \frac{2}{3}$$

$$3 \le 4 + 2\sin^2 x \cos^2 x$$

$$3 \le 2\sqrt{2+\sin^2 x \cos^2 x}$$

$$9 \le 8 + 4\sin^2 x \cos^2 x$$

$$\sin^2 2x \ge -1$$

Equality holds for $x = y = \frac{\pi}{4}$. □

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