# PP35298

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Prove that:

$$\sum_{k=1}^{n} \sqrt{n^2+k^2} \cdot \sin\frac{k\pi}{2n} > \frac{\sqrt{2}n(n+1)}{2}$$

*Solution by Daniel Sitaru.*

$$\sum_{k=1}^{n} \sqrt{n^2+k^2} \cdot \sin\left(\frac{k\pi}{2n}\right) = \sum_{k=1}^{n} \sqrt{2} \cdot \sqrt{\frac{n^2+k^2}{2}} \cdot \sin\left(\frac{k\pi}{2n}\right) >$$

$$\overset{\text{AM-GM}}{>} \frac{\sqrt{2}}{2} \sum_{k=1}^{n} (n+k) \sin\left(\frac{k\pi}{2n}\right) \overset{\text{JORDAN}}{>}$$

$$> \frac{\sqrt{2}}{2} \sum_{k=1}^{n} (n+k) \cdot \frac{2}{\pi} \cdot \frac{k\pi}{2n} =$$

$$= \frac{\sqrt{2}}{2} \sum_{k=1}^{n} \frac{(n+k)k}{n} = \frac{\sqrt{2}}{2} \left( \sum_{k=1}^{n} k + \frac{1}{n} \sum_{k=1}^{n} k^2 \right) =$$

$$= \frac{\sqrt{2}}{2} \left( \frac{n(n+1)}{2} + \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} \right) =$$

$$= \frac{\sqrt{2}n(n+1)}{2} \left( \frac{1}{2} + \frac{2n+1}{3n} \right) > \frac{\sqrt{2}n(n+1)}{2} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2} + \frac{2n+1}{3n} > 1 \Leftrightarrow \frac{2n+1}{3n} > \frac{1}{2}$$

$$\Leftrightarrow 4n+2 > 3n \Leftrightarrow n > -2; n \in \mathbb{N}. \text{ True}$$

□

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