PP35296

MIHÁLY BENCZE - ROMANIA

Prove that:

$$\sum_{k=1}^{n}(n^2+k^2)\Big(\sin\frac{k\pi}{n}\Big)^2 > \frac{n(n+1)(2n+1)}{3}$$

Solution by Daniel Sitaru.

$$\sum_{k=1}^{n}(n^2+k^2)\Big(\sin\frac{k\pi}{n}\Big)^2 \overset{\text{JORDAN}}{>}$$

$$> \sum_{k=1}^{n}(n^2+k^2)\Big(\frac{2}{\pi}\cdot\frac{k\pi}{n}\Big)^2 = 4\sum_{k=1}^{n}\frac{(n^2+k^2)k^2}{n^2} =$$

$$= 4\sum_{k=1}^{n}k^2 + \frac{4}{n^2}\sum_{k=1}^{n}k^4 =$$

$$= \frac{4n(n+1)(2n+1)}{6} + \frac{4}{n^2}\cdot\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} =$$

$$= \frac{n(n+1)(2n+1)}{3}\Big(\frac{4}{2}+\frac{4(3n^2+3n-1)}{10n^2}\Big) > \frac{n(n+1)(2n+1)}{3}$$

$$\Leftrightarrow \frac{4}{2}+\frac{4(3n^2+3n-1)}{10n^2} > 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{4(3n^2+3n-1)}{10n^2} > -1$$

$$\frac{6n^2}{5n^2}+\frac{6n}{5n^2}-\frac{1}{10n^2} > -1$$

$$\frac{6}{5}+\frac{6}{5n}-\frac{1}{10n^2} > -1$$

$$12n-1 > -11n^2$$

$$11n^2+12n > 1 \text{ (True)}$$

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