

PP34291

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\prod_{cyc} \cos \frac{A-B}{2} \geq \frac{s^2 + r^2 + 4Rr}{9R^2}$$

Solution by Daniel Sitaru.

$$\prod_{cyc} \cos \frac{A-B}{2} = \frac{s^2 + r^2 + 2Rr}{8R^2}$$

Remains to prove:

$$\begin{aligned} \frac{s^2 + r^2 + 2Rr}{8R^2} &\geq \frac{s^2 + r^2 + 4Rr}{9R^2} \\ 9s^2 + 9r^2 + 18Rr &\geq 8s^2 + 8r^2 + 32Rr \\ s^2 &\geq 14Rr - r^2 \text{ (to prove)} \\ s^2 &\stackrel{\text{GERRETSEN}}{\geq} 16Rr - 5r^2 \geq 14Rr - r^2 \\ 16Rr - 5r^2 &\geq 14Rr - r^2 \\ 2Rr &\geq 4r^2 \\ R &\geq 2r \end{aligned}$$

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