PP34250

MIHÁLY BENCZE - ROMANIA

In all acute-angled triangles ABC holds:

$$\sum_{cyc} \frac{1}{\sin 2A} \geq \frac{2s}{3r}$$

Solution by Daniel Sitaru.

$$\frac{1}{\sin 2A} + \frac{1}{\sin 2B} + \frac{1}{\sin 2B} \overset{\text{BERGSTRÖM}}{\geq}$$

$$\geq \frac{(1+1+1)^2}{\sin 2A + \sin 2B + \sin 2C} = \frac{9}{4 \sin A \sin B \sin C} =$$

$$= \frac{9}{4 \cdot \frac{a}{2R} \cdot \frac{b}{2R} \cdot \frac{c}{2R}} = \frac{9 \cdot 8R^3}{4abc} =$$

$$= \frac{9 \cdot 2R^3}{4Rrs} = \frac{9R^2}{2rs} \overset{\text{MITRINOVIC}}{\geq}$$

$$\geq \frac{9 \cdot \frac{4s^2}{27}}{2rs} = \frac{\frac{2s}{3}}{r} = \frac{2s}{3r}$$

□

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