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If $1 < x < y$ then:

$$x^{\frac{1}{x-1}} > y^{\frac{1}{y-1}}$$

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$$\forall t > 1: \ln\left(\frac{1}{t}\right) + 1 \leq \frac{1}{t} \Leftrightarrow \frac{1}{t}(t-1) - \ln t \leq 0 \Leftrightarrow \frac{\frac{1}{t}(t-1) - \ln t}{(t-1)^2} \leq 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{d\left(\frac{\ln t}{t-1}\right)}{dt} \leq 0$$

$$1 < x < y \Rightarrow \int_1^x \frac{d\left(\frac{\ln t}{t-1}\right)}{dt} dt > \int_1^y \frac{d\left(\frac{\ln t}{t-1}\right)}{dt} dt \Leftrightarrow \frac{\ln x}{x-1} > \frac{\ln y}{y-1} \Leftrightarrow$$

$$\Leftrightarrow x^{\frac{1}{x-1}} > y^{\frac{1}{y-1}}$$