



SP.036. Let a, b, c be positive real numbers such that

$$3(a + b)(b + c)(c + a) \geq \frac{8}{\sqrt[3]{a^3 + b^3 + c^3}}$$

Prove that $a + b + c \geq \sqrt[3]{9}$.

Proposed by Nguyen Viet Hung – Hanoi – Vietnam

Solution 1 by Anas Adlany - El Jadida- Morroco

Solution 2 by Soumitra Mandal – Kolkata – India

Solution 1 by Anas Adlany - El Jadida- Morroco

We have known that

$$\begin{aligned} (a + b + c)^3 &:= \sum a^3 + 3 \prod (a + b) \\ &\geq \sum a^3 + \frac{8}{\sqrt[8]{a^3 + b^3 + c^3}} \\ &\geq 9 \sqrt[9]{\left(\sum a^3\right) \left(\frac{1}{\sqrt[8]{a^3 + b^3 + c^3}}\right)^8} \end{aligned}$$

Thus,

$$a + b + c \geq \sqrt[3]{9}$$

Hence proved.

Solution 2 by Soumitra Mandal – Kolkata – India

$$\begin{aligned} 3(a + b)(b + c)(c + a) &\geq \frac{8}{\sqrt[3]{a^3 + b^3 + c^3}} \\ \Rightarrow \sum_{cyc} a^3 + 3 \prod_{cyc} (a + b) &\geq \frac{8}{\sqrt[8]{a^3 + b^3 + c^3}} + (a^3 + b^3 + c^3) \end{aligned}$$

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$$\geq (8 + 1)^9 \sqrt{\left\{ \frac{1}{\sqrt[8]{a^3 + b^3 + c^3}} \right\}^8 (a^3 + b^3 + c^3)} = 9$$

$$\Rightarrow (a + b + c)^3 \geq 9 \Rightarrow a + b + c \geq \sqrt[3]{9} \text{ (proved)}$$

$$\text{Equality at } a = b = c = \frac{1}{\sqrt[3]{3}}$$