SOCIETATEA DE ȘTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

Filiala Mehedinți - Mehedinți Branch
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RUSSIAN INEQUALITY – 2

If $a, b, c \in (0, \infty)$ then:

$$\sqrt{\frac{a}{b+c}} + \sqrt{\frac{b}{c+a}} + \sqrt{\frac{c}{a+b}} > 2$$

Proof 1 by Rovsen Pirkuliyev – Sumgait – Azerbaidjian

Proof 2 by Myagmarsuren Yadamsuren – Mongolia

Proof 3 by Marian Dincă – Romania

Proof 1 by Rovsen Pirkuliyev – Sumgait – Azerbaidjian

Denote $a + b + c = x$

Using AM-GM $\Rightarrow$

$$\sqrt{\frac{b+c}{a}} \le \frac{\frac{b+c}{a} + 1}{2} = \frac{x}{2a} \Rightarrow \sqrt{\frac{a}{b+c}} = \frac{2a}{x}$$

$$\sqrt{\frac{c+a}{b}} \le \frac{\frac{c+a}{b} + 1}{2} = \frac{x}{2b} \Rightarrow \sqrt{\frac{b}{c+a}} \ge \frac{2b}{x}$$

and $\sqrt{\frac{c}{a+b}} \ge \frac{2c}{x}$

Hence $\sqrt{\frac{a}{b+c}} + \sqrt{\frac{b}{a+c}} + \sqrt{\frac{c}{a+b}} \ge \frac{2a}{x} + \frac{2b}{x} + \frac{2c}{x} = 2$

Equality is possible

$\frac{b+c}{a} = \frac{a+c}{b} = \frac{a+b}{c} = 1 \Rightarrow$ ***impossible***

$$\sum \sqrt{\frac{a}{b+c}} > 2$$

Proof 2 by Myagmarsuren Yadamsuren – Mongolia

$$\sum \sqrt{\frac{a \cdot (b+c)}{(b+c)^2}} = \sum \frac{\sqrt{a \cdot (b+c)}}{b+c} \geq$$

$$\sqrt{p \cdot q} \geq \frac{2}{\frac{1}{p}+\frac{1}{q}} \geq \sum \frac{\frac{2}{\frac{1}{a}+\frac{1}{b+c}}}{b+c} =$$

$$= \sum \frac{2 \cdot a \cdot (b+c)}{(a+b+c) \cdot (b+c)} = \sum \frac{2a}{a+b+c} = 2$$

Proof 3 by Marian Dincă – Romania

$$\sum_{cyc} \sqrt{\frac{a}{b+c}} = \sum_{cyc} \sqrt{\frac{a^2}{(b+c)a}} = \sum_{cyc} \frac{a}{\sqrt{(b+c)a}} > \sum_{cyc} \frac{a}{\frac{b+c+a}{2}} =$$

$$= \sum_{cyc} \frac{2a}{b+c+a} = 2$$