ROMANIAN MATHEMATICAL MAGAZINE

**UP.632 Find:**

$$\Omega=\int_0^1\int_0^1\int_0^1\frac{x+2y+3z}{e^{x+y+z}}dxdydz$$

*Proposed by Nguyen Hung Cuong – Vietnam*

*Solution 1 by Daniel Sitaru – Romania*

$$\Omega=\int_0^1\int_0^1\int_0^1(x+2y+3z)\,e^{-x-y-z}\,dxdydz=$$

$$=\int_0^1\int_0^1\int_0^1 xe^{-x-y-z}\,dxdydz+2\int_0^1\int_0^1\int_0^1 ye^{-x-y-z}\,dxdydz+$$

$$+3\int_0^1\int_0^1\int_0^1 ze^{-x-y-z}\,dxdydz=6\int_0^1\int_0^1\int_0^1 xe^{-x}e^{-y}e^{-z}\,dxdydz$$

$$=6\left(\int_0^1 xe^{-x}\,dx\right)\left(\int_0^1 e^{-y}dy\right)\left(\int_0^1 e^{-z}dz\right)=$$

$$=6\left(-xe^{-x}|_0^1+\int_0^1 e^{-x}dx\right)\left(-e^{-y}|_0^1\right)\left(-e^{-z}|_0^1\right)=$$

$$=6\left(-\frac{1}{e}-\frac{1}{e}+e^0\right)(-e^{-1}+e^0)^2=6\left(1-\frac{2}{e}\right)\left(1-\frac{1}{e}\right)^2$$

*Solution 2 by Jose Luis Diaz Barrero-Spain*

Since $e^{-(x+y+z)}=e^{-x}e^{-y}e^{-z}$, the integrand separates. We need the two one-variable integrals

$$S=\int_0^1 e^{-t}\,dt=1-\frac{1}{e},\qquad B=\int_0^1 te^{-t}=[-(t+1)e^{-t}]_0^1=1-\frac{2}{e}.$$

Each term of the numerator gives a product of one-variable integrals. For instance,

$$\int_0^1\int_0^1\int_0^1 xe^{-x}e^{-y}e^{-z}\,dx\,dy\,dz=BA^2,$$

and the terms in $y$ and $z$ give the same value by symmetry. Therefore

$$\Omega=(1+2+3)BA^2=6\left(1-\frac{2}{e}\right)\left(1-\frac{1}{e}\right)^2=\frac{6(e-2)(e-1)^2}{e^3}.$$