ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^1\int_0^1\int_0^1\sqrt[3]{x+2y+3z}\,dxdydz$$

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$$step-1:$$

$$\int_0^1\sqrt[3]{x+2y+3z}dx=\left(\frac{3}{4}(x+2y+3z)^{\frac{4}{3}}\right)_0^1=\frac{3}{4}\left((2y+3z+1)^{\frac{4}{3}}-(2y+3z)^{\frac{4}{3}}\right)$$

$$step-2$$

$$\frac{3}{4}\left(\int_0^1(2y+3z+1)^{\frac{4}{3}}dy-\int_0^1(2y+3z)^{\frac{4}{3}}dy\right)=$$

$$=\frac{3}{4}\left(\frac{3}{7}.\frac{1}{2}(2y+3z+1)^{\frac{7}{3}}-\frac{1}{2}.\frac{3}{7}(2y+3z)^{\frac{7}{3}}\right)_0^1=$$

$$=\frac{9}{56}\left((3z+3)^{\frac{7}{3}}-(3z+2)^{\frac{7}{3}}-(3z+1)^{\frac{7}{3}}+(3z)^{\frac{7}{3}}\right)$$

$$step\ 3:$$

$$\frac{9}{56}\int_0^1\left((3z+3)^{\frac{7}{3}}-(3z+2)^{\frac{7}{3}}-(3z+1)^{\frac{7}{3}}+(3z)^{\frac{7}{3}}\right)dz=$$

$$=\frac{9}{56}\left(\frac{3}{10}.\frac{1}{3}(3z+3)^{\frac{10}{3}}-\frac{1}{3}.\frac{3}{10}(3z+2)^{\frac{10}{3}}-\frac{1}{3}.\frac{3}{10}(3z+1)^{\frac{10}{3}}+\frac{1}{3}.\frac{3}{10}(3z)^{\frac{10}{3}}\right)_0^1=$$

$$=\frac{9}{560}\left(6^{\frac{10}{3}}-5^{\frac{10}{3}}-4^{\frac{10}{3}}+2^{\frac{10}{3}}+1\right)=$$

$$\int_0^1\int_0^1\int_0^1\sqrt[3]{x+2y+3z}\,dxdydz=\frac{9}{560}\left(6^{\frac{10}{3}}-5^{\frac{10}{3}}-4^{\frac{10}{3}}+2^{\frac{10}{3}}+1\right)$$