

ROMANIAN MATHEMATICAL MAGAZINE

If $0 < a \leq b$ then:

$$\int_a^b \int_a^b \frac{dxdy}{\sqrt{x^2 + y^2}} \leq \frac{1}{\sqrt{2}} (b - a)(\ln b - \ln a)$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\begin{aligned} \int_a^b \int_a^b \frac{dxdy}{\sqrt{x^2 + y^2}} &\stackrel{AM-GM}{\leq} \int_a^b \int_a^b \frac{dxdy}{\sqrt{2xy}} = \frac{1}{\sqrt{2}} \int_a^b \int_a^b \sqrt{\frac{1}{x} \cdot \frac{1}{y}} dxdy \stackrel{AM-GM}{\leq} \\ &\stackrel{AM-GM}{\leq} \frac{1}{2\sqrt{2}} \int_a^b \int_a^b \left(\frac{1}{x} + \frac{1}{y} \right) dxdy = \frac{1}{2\sqrt{2}} \left(\int_a^b \int_a^b \frac{1}{x} dxdy + \int_a^b \int_a^b \frac{1}{y} dxdy \right) \\ &= \frac{1}{2\sqrt{2}} \left((\ln x)_a^b (y)_a^b + (\ln y)_a^b (x)_a^b \right) = \frac{1}{\sqrt{2}} (b - a)(\ln b - \ln a) \end{aligned}$$

Equality holds for $a = b$