ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\infty} \frac{dx}{(1+x^2)\left(1+\sqrt[3]{x}\right)}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$Let\ I = \int_0^{\infty} \frac{dx}{(1+x^2)\left(1+\sqrt[3]{x}\right)} \quad (1)$$

$$Put\ x = \frac{1}{t}\ then\ dx = -\frac{1}{t^2}dt$$

$$then\ I = \int_0^{\infty} \frac{\sqrt[3]{t}\,dx}{(1+t^2)\left(1+\sqrt[3]{t}\right)} = \int_0^{\infty} \frac{\sqrt[3]{x}\,dx}{(1+x^2)\left(1+\sqrt[3]{x}\right)} (2)$$

$$adding\ (1)\ \&\ (2) we\ get:$$

$$2I = \int_0^{\infty} \frac{\left(1+\sqrt[3]{x}\right)dx}{(1+x^2)\left(1+\sqrt[3]{x}\right)} = \int_0^{\infty} \frac{dx}{(1+x^2)} = (\arctan x)_0^{\infty} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$