

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{x}{x^3 + 1} dx$$

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Solution by Daniel Sitaru – Romania

We will use the formula:

$$\begin{aligned} \int_0^{\infty} \frac{x^m}{1+x^n} dx &= \frac{\pi}{n \sin\left(\frac{m+1}{n} \cdot \pi\right)}; 0 < m+1 < n \\ \Omega &= \int_0^{\infty} \frac{x}{x^3+1} dx = \frac{\pi}{3 \sin\left(\frac{1+1}{3} \cdot \pi\right)} = \frac{\pi}{3 \sin\left(\frac{2\pi}{3}\right)} = \\ &= \frac{\pi}{3 \sin\left(\pi - \frac{\pi}{3}\right)} = \frac{\pi}{3 \sin\frac{\pi}{3}} = \frac{\pi}{3 \cdot \frac{\sqrt{3}}{2}} = \frac{2\pi}{3\sqrt{3}} = \frac{2\pi\sqrt{3}}{3 \cdot 3} = \frac{2\sqrt{3}\pi}{9} \end{aligned}$$