ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt{x}dx}{x^2\sqrt{x}+1}$$

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Solution by Daniel Sitaru – Romania

We will use the formula:

$$\int_0^{\infty} \frac{x^m}{1+x^n} = \frac{\pi}{n\sin\left(\frac{m+1}{n}\pi\right)}; 0 < m+1 < n$$

$$\Omega = \int_0^{\infty} \frac{\sqrt{x}}{x^2\sqrt{x}+1}dx = \int_0^{\infty} \frac{x^{\frac{1}{2}}}{x^{2+\frac{1}{2}}+1}dx = \int_0^{\infty} \frac{x^{\frac{1}{2}}}{x^{\frac{5}{2}}+1}dx = \frac{\pi}{\frac{5}{2}\sin\left(\frac{\frac{1}{2}+1}{\frac{5}{2}}\pi\right)} =$$

$$= \frac{2\pi}{5\sin\left(\frac{3\pi}{5}\right)} = \frac{2\pi}{5\sin\left(\pi-\frac{3\pi}{5}\right)} = \frac{2\pi}{5\sin\frac{2\pi}{5}} = \frac{8\pi}{5\sqrt{10+2\sqrt{5}}}$$