

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{x^4}{1+x^6} dx$$

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Solution by Daniel Sitaru – Romania

We will use the formula:

$$\begin{aligned} \int_0^{\infty} \frac{x^m}{1+x^n} dx &= \frac{\pi}{n \sin\left(\frac{m+1}{n} \cdot \pi\right)}; 0 < m+1 < n \\ \Omega = \int_0^{\infty} \frac{x^4}{1+x^6} dx &= \frac{\pi}{6 \sin\left(\frac{4+1}{6} \pi\right)} = \frac{\pi}{6 \sin\left(\frac{5\pi}{6}\right)} = \frac{\pi}{6 \sin\left(\pi - \frac{\pi}{6}\right)} = \\ &= \frac{\pi}{6 \sin \frac{\pi}{6}} = \frac{\pi}{6 \cdot \frac{1}{2}} = \frac{\pi}{3} \end{aligned}$$