ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt[3]{x}}{x^4+1}\,dx$$

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Solution by Daniel Sitaru – Romania

We will use the formula:

$$\int_0^{\infty} \frac{x^m}{1+x^n}\,dx = \frac{\pi}{n\sin\left(\frac{m+1}{n}\pi\right)}; 0 < m+1 < n$$

$$\Omega = \int_0^{\infty} \frac{\sqrt[3]{x}}{x^4+1}\,dx = \int_0^{\infty} \frac{x^{\frac{1}{3}}}{1+x^4}\,dx =$$

$$= \frac{\pi}{4\sin\left(\frac{\frac{1}{3}+1}{4}\pi\right)} = \frac{\pi}{4\sin\frac{\pi}{3}} = \frac{\pi}{4\cdot\frac{\sqrt{3}}{2}} = \frac{\pi}{2\sqrt{3}} = \frac{\pi\sqrt{3}}{6}$$