

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt[4]{x}}{x^2 + 1} dx$$

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Solution by Daniel Sitaru – Romania

We will use the formula:

$$\int_0^{\infty} \frac{x^m}{1+x^n} dx = \frac{\pi}{n \sin\left(\frac{m+1}{n}\pi\right)}; 0 < m+1 < n$$

$$\Omega = \int_0^{\infty} \frac{\sqrt[4]{x}}{1+x^2} dx = \int_0^{\infty} \frac{x^{\frac{1}{4}}}{1+x^2} dx = \frac{\pi}{2 \sin\left(\frac{\frac{1}{4}+1}{2}\pi\right)} = \frac{\pi}{2 \sin\left(\frac{5\pi}{8}\right)} = \frac{\pi}{\sqrt{2+\sqrt{2}}}$$