ROMANIAN MATHEMATICAL MAGAZINE

**Find:**

$$\Omega = \int_0^{\infty} \frac{\sqrt[3]{x}}{x^3+1}\,dx$$

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**Denote:** $x^3 = y \Rightarrow x = \sqrt[3]{y} \Rightarrow dx = \frac{1}{3\sqrt[3]{y^2}}dy$

$$\Omega = \frac{1}{3}\int_0^{\infty} \frac{y^{\frac{1}{9}-\frac{2}{3}}}{y+1}dy = \frac{1}{3}\int_0^{\infty} \frac{y^{\frac{1-6}{9}}}{y+1}dy = \frac{1}{9}\int_0^{\infty} \frac{y^{-\frac{5}{9}}}{y+1}dy = \frac{1}{9}\int_0^{\infty} \frac{y^{\frac{4}{9}-1}}{y+1}dy =$$

$$= \frac{1}{9}\cdot\frac{\pi}{\sin\left(\frac{4\pi}{9}\right)} = \frac{\pi}{9\sin\left(\frac{4\pi}{9}\right)}$$

**We used the formula:**

$$\int_0^{\infty} \frac{t^{p-1}}{1+t}dt = \frac{\pi}{\sin(\pi p)}; 0 < p < 1$$