ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt{x}}{x^4+1}\,dx$$

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Denote:

$$x^4 = y \Rightarrow x = \sqrt[4]{y} \Rightarrow dx = \frac{1}{4\sqrt[4]{y^3}}dy$$

$$\Omega = \int_0^{\infty} \frac{y^{\frac{1}{8}} \cdot \frac{1}{4} \cdot y^{-\frac{3}{4}}}{y+1}dy = \frac{1}{4}\int_0^{\infty} \frac{y^{\frac{1}{8}-\frac{3}{4}}}{y+1}dy = \frac{1}{4}\int_0^{\infty} \frac{y^{-\frac{5}{8}}}{y+1}dy = \frac{1}{4}\int_0^{\infty} \frac{y^{\frac{3}{8}-1}}{y+1}dy =$$

$$= \frac{1}{4} \cdot \frac{\pi}{\sin\left(3 \cdot \frac{\pi}{8}\right)} = \frac{\pi}{4\sin\left(\frac{3\pi}{8}\right)} = \frac{\pi}{2\sqrt{2+\sqrt{2}}}$$

We used the formula:

$$\int_0^{\infty} \frac{t^{p-1}}{1+t}dt = \frac{\pi}{\sin(\pi p)}; 0 < p < 1$$