

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt[4]{x}}{x^2 + 1} dx$$

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Denote:

$$\begin{aligned} x^2 = y &\Rightarrow x = \sqrt{y} \Rightarrow dx = \frac{1}{2\sqrt{y}} dy \\ \Omega &= \int_0^{\infty} \frac{y^{\frac{1}{8}}}{y+1} \cdot \frac{1}{2\sqrt{y}} dy = \frac{1}{2} \int_0^{\infty} \frac{y^{\frac{1}{8}-\frac{1}{2}}}{y+1} dy = \frac{1}{2} \int_0^{\infty} \frac{y^{-\frac{3}{8}}}{y+1} dy = \frac{1}{2} \int_0^{\infty} \frac{y^{\frac{5}{8}-1}}{y+1} dy = \\ &= \frac{1}{2} \cdot \frac{\pi}{\sin\left(\frac{5\pi}{8}\right)} = \frac{\pi}{2 \sin\left(\frac{5\pi}{8}\right)} = \frac{\pi}{\sqrt{2} + \sqrt{2}} \end{aligned}$$

We used the formula:

$$\int_0^{\infty} \frac{t^{p-1}}{1+t} dt = \frac{\pi}{\sin(\pi p)}; 0 < p < 1$$