

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{x^3}{x^5 + 1} dx$$

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$$\begin{aligned}\Omega &= \int_0^{\infty} \frac{x^3}{x^5 + 1} dx = \frac{\pi}{5 \sin\left(\frac{3+1}{5} \cdot \pi\right)} = \\ &= \frac{\pi}{5 \sin\left(\frac{4\pi}{5}\right)} = \frac{\pi}{5 \sin\left(\pi - \frac{\pi}{5}\right)} = \frac{\pi}{5 \sin\frac{\pi}{5}} = \frac{\pi}{5 \cdot \frac{\sqrt{10-2\sqrt{5}}}{4}} = \frac{4\pi}{5\sqrt{10-2\sqrt{5}}}\end{aligned}$$

We used the formula:

$$\int_0^{\infty} \frac{x^m}{x^n + 1} dx = \frac{\pi}{n \sin\left(\frac{m+1}{n} \cdot \pi\right)}; 0 < m+1 < n$$