

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt{x}}{x^2 + 1} dx$$

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$$\begin{aligned}\Omega &= \int_0^{\infty} \frac{\sqrt{x}}{x^2 + 1} dx = \int_0^{\infty} \frac{x^{\frac{1}{2}}}{x^2 + 1} dx = \frac{\pi}{2 \sin\left(\left(\frac{1}{2} + 1\right) \cdot \frac{\pi}{2}\right)} = \frac{\pi}{2 \sin\left(\frac{3\pi}{4}\right)} = \\ &= \frac{\pi}{2 \sin\left(\pi - \frac{\pi}{4}\right)} = \frac{\pi}{2 \sin\frac{\pi}{4}} = \frac{\pi}{2 \cdot \frac{\sqrt{2}}{2}} = \frac{\pi}{\sqrt{2}} = \frac{\pi\sqrt{2}}{2}\end{aligned}$$

We used the formula:

$$\int_0^{\infty} \frac{x^m}{x^n + 1} dx = \frac{\pi}{n \sin\left(\frac{m+1}{n} \cdot \pi\right)}; 0 < m+1 < n$$