

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt{x}}{x^4\sqrt{x}+1} dx$$

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We will use the formula:

$$\begin{aligned} \int_0^{\infty} \frac{x^m}{x^n+1} dx &= \frac{\pi}{n \sin\left(\frac{m+1}{n}\pi\right)}; 0 < m+1 < n \\ \Omega = \int_0^{\infty} \frac{\sqrt{x}}{x^4\sqrt{x}+1} dx &= \int_0^{\infty} \frac{x^{\frac{1}{2}}}{x^{\frac{9}{2}}+1} dx = \frac{\pi}{\frac{9}{2} \sin\left(\frac{\frac{1}{2}+1}{\frac{9}{2}}\pi\right)} = \frac{2\pi}{9 \sin\left(\frac{\pi}{3}\right)} = \\ &= \frac{2\pi}{9 \cdot \frac{\sqrt{3}}{2}} = \frac{4\pi}{9\sqrt{3}} = \frac{4\pi\sqrt{3}}{27} \end{aligned}$$