

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt[3]{x}}{x^2 \sqrt[3]{x} + 1} dx$$

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We will use the formula:

$$\int_0^{\infty} \frac{x^m}{1+x^n} dx = \frac{\pi}{n \sin\left(\frac{m+1}{n}\pi\right)}; 0 < m+1 < n$$

$$\begin{aligned}\Omega &= \int_0^{\infty} \frac{\sqrt[3]{x}}{x^2 \cdot \sqrt[3]{x} + 1} dx = \int_0^{\infty} \frac{x^{\frac{1}{3}}}{x^2 \cdot x^{\frac{1}{3}} + 1} dx = \int_0^{\infty} \frac{x^{\frac{1}{3}}}{x^{2+\frac{1}{3}} + 1} dx = \int_0^{\infty} \frac{x^{\frac{1}{3}}}{x^{\frac{7}{3}} + 1} dx = \\ &= \frac{\pi}{\frac{7}{3} \sin\left(\frac{\frac{1}{3} + 1}{\frac{7}{3}}\pi\right)} = \frac{3\pi}{7 \sin\left(\frac{4\pi}{7}\right)} = \frac{3\pi}{7 \sin\left(\pi - \frac{4\pi}{7}\right)} = \frac{3\pi}{7 \sin\left(\frac{3\pi}{7}\right)}\end{aligned}$$