

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{\sqrt{x}}{x^2\sqrt{x} + 1} dx$$

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Solution by Daniel Sitaru – Romania

We will use the formula:

$$\begin{aligned} \int_0^{\infty} \frac{x^m}{1+x^n} dx &= \frac{\pi}{n \sin\left(\frac{m+1}{n}\pi\right)}; 0 < m+1 < n \\ \Omega &= \int_0^{\infty} \frac{\sqrt{x}}{x^3\sqrt{x}+1} dx = \int_0^{\infty} \frac{x^{\frac{1}{2}}}{x^3 \cdot x^{\frac{1}{2}} + 1} dx = \int_0^{\infty} \frac{x^{\frac{1}{2}}}{x^{\frac{7}{2}} + 1} dx = \\ &= \frac{\pi}{\frac{7}{2} \sin\left(\frac{\frac{1}{2}+1}{\frac{7}{2}}\pi\right)} = \frac{2\pi}{7 \sin\left(\frac{3\pi}{7}\right)} \end{aligned}$$