

Number 43

WINTER 2026

R M M

ROMANIAN MATHEMATICAL MAGAZINE

Founding Editor
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Available online
www.ssmrmh.ro

ISSN-L 2501-0099

PROBLEMS FOR JUNIORS

JP.631. Let a, b, c, d be nonnegative real numbers such that $a \geq b \geq c \geq d \geq \frac{1}{2}$ and $ab + bc + cd + da = 4$. Prove that:

$$\frac{1}{a+3} + \frac{1}{b+3} + \frac{1}{c+3} + \frac{1}{d+3} \leq \frac{64}{63}$$

Proposed by Vasile Cîrtoaje - Romania

JP.632. Let a, b, c, d, e be nonnegative real numbers such that $a \geq b \geq c \geq d \geq e$ and $(a+b)(c+d+e) = 6$. Prove that:

$$\frac{1}{2a+3} + \frac{1}{2b+3} + \frac{1}{2c+3} + \frac{1}{2d+3} + \frac{1}{2e+3} \geq 1$$

Proposed by Vasile Cîrtoaje - Romania

JP.633. If $a, b, c \geq 1; a^2 + b^2 + c^2 = 3$ then:

$$(a^5 + 10a^3 + 5a - 1)(b^5 + 10b^3 + 5b - 1)(c^5 + 10c^3 + 5c - 1) \geq 3375$$

Proposed by Daniel Sitaru - Romania

JP.634. Find $x, y \in (0, \pi)$ such that:

$$\frac{\sin^2 x}{1 + \sin^2 x} + \frac{\sin^2 y}{(1 + \sin^2 x)(1 + \sin^2 y)} + \frac{1}{4 \sin x \sin y} = 1$$

Proposed by Daniel Sitaru - Romania

JP.635. If $a, b > 0; ab = 1$ then:

$$\frac{a}{3a^2 + b} + \frac{b}{3b^2 + a} \leq \frac{1}{2}$$

Proposed by Nguyen Hung Cuong - Vietnam

JP.636. If $a, b > 0$ and $ab = 1$ then:

$$\frac{a}{\sqrt{a^2 + 1}} + \frac{b}{\sqrt{b^2 + 1}} \leq \sqrt{2}$$

Proposed by Nguyen Hung Cuong - Vietnam

JP.637. Solve for positive real numbers the following system of equations:

$$\begin{aligned}x^2 + y^2 &= 1, \\ 1 + 2 \cdot (3x - 4x^3) \cdot (3y - 4y^3) &= 0\end{aligned}$$

Proposed by Neculai Stanciu - Romania

JP.638. If $a, b, c > 0$, prove that $a + b + c \leq \frac{a^3}{bc} + \frac{b^3}{ca} + \frac{c^3}{ab}$.

Proposed by Neculai Stanciu - Romania

JP.639. If $x, y, z > 0, \sum x = 1$ and $\lambda \leq 4$ then:

$$\lambda \sum x^3 + 3(\lambda + 1)xyz \geq \frac{2\lambda + 1}{9}$$

Proposed by Marin Chirciu - Romania

JP.640. If $x, y, z > 0, x + y + z = \frac{\pi}{2}$ then:

$$\sum \left(\frac{\cos(y - z)}{\sin x} \right)^n \geq 3 \cdot 2^n$$

Proposed by Marin Chirciu - Romania

JP.641. Solve for real numbers:

$$4\sqrt{x - 4} + 10\sqrt{y - 25} + 14\sqrt{z - 49} = x + y + z$$

Proposed by Daniel Sitaru - Romania

JP.642. If $a, b, c > 0, \sum a = 3$ and $n \in \mathbb{N}$ then:

$$\sum \left(\frac{a^3}{a^2 + ab + b^2} \right)^n \geq \frac{3}{3^n}$$

Proposed by Marin Chirciu - Romania

JP.643. Let $\lambda > 0$ fixed. If $x, y, z > 0, x^2 + y^2 + z^2 = \lambda$ then find the maximum value of

$$P = \frac{x}{\lambda - yz} + \frac{y}{\lambda - zx} + \frac{z}{\lambda - xy}$$

Proposed by Marin Chirciu - Romania

JP.644. Solve for real numbers:

$$\sqrt{x^2 + (4 - x)^2} + \sqrt{y^2 + (8 - y)^2} = 6\sqrt{2}$$

Proposed by Daniel Sitaru - Romania

JP.645. Solve for real numbers:

$$\begin{cases} x, y, z > 0 \\ x + y + z = 1 \\ x\sqrt{3y} + y\sqrt{3z} + z\sqrt{3x} = 1 \end{cases}$$

Proposed by Daniel Sitaru - Romania

PROBLEMS FOR SENIORS

SP.631. Let a, b, c be nonnegative real numbers such that at most one of them is larger than 1 and $ab + bc + ca = 3$. Prove that:

$$a^{\frac{2}{3}} + b^{\frac{2}{3}} + c^{\frac{2}{3}} \geq 3$$

Proposed by Vasile Cîrtoaje - Romania

SP.632. Prove that 2 is the largest positive value of the constant k such that:

$$a^2 + b^2 + c^2 + d^2 - 4 \geq k(ab + bc + cd + da - 4)$$

for all positive real numbers a, b, c, d with at most one of them larger than 1 and $abcd = 1$.

Proposed by Vasile Cîrtoaje - Romania

SP.633. Let $n \geq 3$ be an integer, and let $a_1, a_2, \dots, a_n$ be positive real numbers such that

$$\sum_{r=1}^n a_r^2 = n.$$

For every triple $1 \leq i < j < k \leq n$, put

$$R_{ijk} = \frac{a_i}{3a_i + a_j} + \frac{a_j}{3a_j + a_k} + \frac{a_k}{3a_k + a_i}$$

and

$$D_{ijk} = \sqrt{\frac{3(a_i^2 + a_j^2 + a_k^2)}{a_i + a_j + a_k}}$$

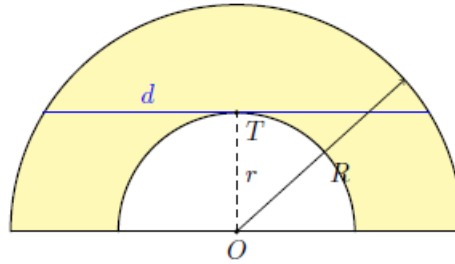
Prove that

$$\frac{1}{\binom{n}{3}} \sum_{1 \leq i < j < k \leq n} \left(\frac{2}{3} R_{ijk} + \frac{3}{2} D_{ijk} \right) \geq 2$$

Determine all cases of equality.

Proposed by Mihály Bencze - Romania

SP.634. Let two concentric spheres have radii R and r , where $R > r > 0$, and let their common centre be O . A plane parallel to their common equatorial plane is tangent to the smaller sphere in the upper half-space. Its intersection with the larger sphere is a circle of diameter d . The two upper solid hemispheres bound a hemispherical shell. Determine the volume V of this shell in terms of the general data r and d . Also determine the difference S of the curved surface areas of the two hemispheres.



The figure is a meridian section of the three-dimensional configuration.

Proposed by Mihály Bencze - Romania

SP.635. Let $n \geq 2$, and let $a_1, a_2, \dots, a_n$ be positive real numbers whose sum is

$$S = \sum_{i=1}^n a_i$$

Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be positive real numbers satisfying

$$\sum_{i=1}^n \lambda_i = 1$$

Prove that

$$\sum_{i=1}^n \lambda_i \frac{a_i}{S - a_i} \geq \frac{S}{\prod_{i=1}^n (S - a_i)^{\lambda_i}} - 1.$$

Determine when equality holds.

Proposed by Mihály Bencze - Romania

SP.636. Find all positive real numbers such that $ab + bc + ca = 3abc$, and

$$\sqrt{\frac{a+b}{a^2+b^2}} + \sqrt{\frac{b+c}{b^2+c^2}} + \sqrt{\frac{c+a}{c^2+a^2}} = 3.$$

Proposed by José Luis Díaz - Barrero - Spain

SP.637. If $a, b, c > 0$, $a + b + c = 3$ and $\lambda \geq 0$ then:

$$\frac{a + \lambda}{b^2 + 1} + \frac{b + \lambda}{c^2 + 1} + \frac{c + \lambda}{a^2 + 1} \geq \frac{3(\lambda + 1)}{2}$$

Proposed by Marin Chirciu - Romania

SP.638. If $x, y, z \geq 1$ and $\lambda \geq 0$ then:

$$\frac{x^3 + x + \lambda}{y + z + \lambda} + \frac{y^3 + y + \lambda}{z + x + \lambda} + \frac{z^3 + z + \lambda}{x + y + \lambda} \geq 3$$

Proposed by Marin Chirciu - Romania

SP.639. If $a, b, c > 0$ and $1 \leq \lambda \leq 2$ then:

$$\sum \frac{xy}{x^2 + \lambda y^2} \leq \frac{3}{\lambda + 1}$$

Proposed by Marin Chirciu - Romania

SP.640. Find all prime numbers p, q, r with $q < 32$ such that:

$$p^q + q^p = r(4p + 1).$$

Proposed by José Luis Díaz - Barrero - Spain

SP.641. Let $A(z)$ be a polynomial with complex coefficients, and let $a, b, c \in \mathbb{C}$. Suppose that $A(z)$ leaves the same remainder upon division by $(z - a)(z - b)$ and upon division by $(z - a)(z - c)$. Determine the value of

$$(a - b)A(c) + (b - c)A(a) + (c - a)A(b)$$

Proposed by José Luis Díaz - Barrero - Spain

SP.642. It is possible to write each positive integer as a sum of distinct Lucas numbers such that no two summands are neighbors in the Lucas sequence? The same question for Fibonacci numbers.

Proposed by José Luis Díaz - Barrero - Spain

SP.643. Find the equation of the common perpendicular line and the minimum distance between the lines r and s , given by

$$r : \begin{cases} 2x - y - 2 = 0 \\ y - 2z + 6 = 0 \end{cases} \quad s : \begin{cases} x - y - 2 = 0 \\ 2y - z + 1 = 0 \end{cases}$$

Proposed by José Luis Díaz - Barrero - Spain

SP.644. Let a, b, c, d be nonnegative real numbers such that $a \geq b \geq 1 \geq c \geq d$ and

$$ab + ac + ad + bc + bd + cd = 6$$

Prove that:

$$\frac{1}{2a+1} + \frac{1}{2b+1} + \frac{1}{2c+1} + \frac{1}{2d+1} \geq \frac{a+b+c+d}{3}$$

Proposed by Vasile Cîrtoaje - Romania

SP.645. Under stereographic projection, what condition must two points z_1, z_2 in the plane satisfy for them to correspond to a pair of antipodal (diametrically opposite) points on the sphere?

Proposed by José Luis Díaz - Barrero - Spain

UNDERGRADUATE PROBLEMS

UP.631. Prove that:

$$\int_0^1 \frac{(\log x)^2}{x^2 - x - 1} dx = \frac{2}{2\varphi - 1} \left(Li_3(-\varphi) - Li_3\left(\frac{1}{\varphi}\right) \right)$$

where $\varphi = \frac{1+\sqrt{5}}{2}$ is the Golden Ratio.

Proposed by Omkumar Rao - India

UP.632. Find:

$$\Omega = \int_0^1 \int_0^1 \int_0^1 \frac{x + 2y + 3z}{e^{x+y+z}} dx dy dz$$

Proposed by Nguyen Hung Cuong - Vietnam

UP.633. Prove the equality:

$$\int_0^\infty \frac{\arctan x \cdot |\ln x|}{x^2 - x + 1} dx = \frac{\pi}{30} \left[\psi_1\left(\frac{1}{6}\right) - 2\pi^2 \right]$$

where $\psi_1(x)$ is trigamma function.

Proposed by Vasile Mircea Popa - Romania

UP.634. Calculate the integral:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \operatorname{arccot} x (|\sin x|)^{\frac{1}{3}} (\cos x)^{\frac{5}{3}} dx$$

In this problem we will consider that definition of the function $\operatorname{arccot}(x)$ which has the image the interval $(0, \pi)$.

Proposed by Vasile Mircea Popa - Romania

UP.635. Prove that:

$$\int_0^{\frac{\pi}{2}} (\cos x)^2 e^{\cos(\tan(x))} \cos(\sin(\tan(x))) dx = \frac{\pi(e+1)}{4e} \sqrt{e}$$

Proposed by Omkumar Rao - India

UP.636. Solve the linear differential equation

$$y^{IV} - 5y''' + 10y'' - 10y' + 4y = x^4 - 5x^3 + 10x^2 - 10x + 4$$

Proposed by José Luis Díaz - Barrero - Spain

UP.637. Find the general solution of the system

$$x' = x + 2y + 3z,$$

$$y' = 3x + y + 2z,$$

$$z' = 2x + 3y + z.$$

Proposed José Luis Díaz - Barrero - Spain

UP.638. Compute the square root of the following matrix

$$X = \frac{1}{14} \begin{pmatrix} 41 & 23 & -25 & 1 \\ 19 & 46 & -29 & 2 \\ 21 & -7 & 7 & 7 \\ 17 & -6 & -23 & 46 \end{pmatrix}$$

Proposed by José Luis Díaz - Barrero - Spain

UP.639. Find:

$$\Omega = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\sum_{i=1}^k \left(i^2 + \frac{i}{3} - \frac{1}{12} \right) \right]^{-1}$$

Proposed by Omkumar Rao - India

UP.640. Compute the following sum

$$\sum_{n=1}^{\infty} \frac{x^n}{1 - x^n - x^{n+1} + x^{2n+1}}$$

Proposed by José Luis Díaz - Barrero - Spain

UP.641. Find the integral of the total differentials:

$$\Omega = \int_{(3,4,5)}^{(6,8,10)} \frac{xdx + ydy + zdz}{\sqrt{x^2 + y^2 + z^2}}$$

Proposed by Daniel Sitaru - Romania

UP.642. Find the line integral of the total differentials:

$$\Omega = \int_{(0,0,0)}^{(\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2})} 2 \cos x dx + 3 \cos y dy + 5 \cos z dz$$

Proposed by Daniel Sitaru - Romania

UP.643. Find:

$$\Omega = \int_0^\infty \int_0^\infty \int_0^\infty (2x + 3y + 5z) e^{-x^2 - y^2 - z^2} dx dy dz$$

Proposed by Daniel Sitaru - Romania

UP.644. Let be:

$$D = \{(x, y, z) | x, y, z \in \mathbb{R}; 0 \leq x \leq 1; 0 \leq y \leq x; 0 \leq z \leq xy\}$$

Find:

$$\Omega = \iiint_D xyz dx dy dz$$

Proposed by Daniel Sitaru - Romania

UP.645. Find:

$$\Omega = \int_0^1 \int_0^1 \int_0^1 \frac{e^{2x} + e^{3y} + e^{4z}}{e^{x+y+z}} dx dy dz$$

Proposed by Nguyen Hung Cuong - Vietnam

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