ROMANIAN MATHEMATICAL MAGAZINE CHALLENGES III

DANIEL SITARU - ROMANIA

01. Solve for real numbers:

$$\sqrt{4+x}+\sqrt{4-x}+\sqrt{16-x^2}=8$$

Proposed by Marin Chirciu - Romania

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} a=\sqrt{4+x} \\ b=\sqrt{4-x} \end{cases}$; $\begin{cases} S=a+b>0 \\ P=ab>0 \end{cases}$

$$\Rightarrow \begin{cases} a+b+ab=8 \\ a^2+b^2=8 \end{cases} \Rightarrow \begin{cases} S+P=8 \\ S^2-2P=8 \end{cases} \Rightarrow P=8-s$$

$$S^2-2(8-S)=8 \Rightarrow S^2-16+2S-8=0$$

$$S^2+2S-24=0 \Rightarrow \Delta=100$$

$$S=\frac{-2+10}{2}=4 \Rightarrow P=8-4=4$$

$$\begin{cases} S=4 \\ P=4 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=2 \end{cases} \Rightarrow \begin{cases} \sqrt{4+x}=2 \\ \sqrt{4-x}=2 \end{cases} \Rightarrow x=0$$

□

02. If $a,b>0, a+b=2$ then:

$$\frac{1}{a^3}+\frac{1}{b^3}+12ab\geq 14$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} S=a+b \\ P=ab \end{cases} \Rightarrow \begin{cases} S=2 \\ a^3+b^3=S^3-3SP=8-6P \end{cases}$

$$\frac{1}{a^3}+\frac{1}{b^3}+12ab\geq 14 \Leftrightarrow \frac{a^3+b^3}{a^3b^3}+12ab\geq 14$$

$$\Leftrightarrow \frac{S^3-3SP}{P^3}+12P\geq 14 \Leftrightarrow \frac{8-6P}{P^3}+12P\geq 14$$

$$8-6P+12P^4\geq 14P^3$$

$$12P^4-14P^3-6P+8\geq 0$$

$$6P^4-7P^3-3P+4\geq 0$$

$$6P^4-6P^3-P^3+P^2-P^2+P-4P+4\geq 0$$

$$6P^3(P-1)-P^2(P-1)-P(P-1)-4(P-1)\geq 0$$

$$(P-1)(6P^3-P^2-P-4)\geq 0$$

$$(P-1)[(6P^3-6P^2)+(5P^2-5P)+(4P-4)]\geq 0$$
$$(P-1)[6P^2(P-1)+5P(P-1)+4(P-1)]\geq 0$$
$$(P-1)^2(6P^2+5P+4)\geq 0 \qquad \text{(true)}$$

Equality holds for $a=b=1$. $\square$

03. If $a,b>0; a+b=2$ then:

$$\frac{1}{a^2}+\frac{1}{b^2}+6ab\geq 8$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} S=a+b \\ P=ab \end{cases} \Rightarrow \begin{cases} S=2 \\ a^2+b^2=S^2-2P=4-2P \end{cases}$

$$\frac{1}{a^2}+\frac{1}{b^2}+6ab\geq 8 \Leftrightarrow \frac{a^2+b^2}{a^2b^2}+6ab\geq 8$$
$$\frac{4-2P}{P^2}+6P\geq 8 \Leftrightarrow 4-2P+6P^3-8P^2\geq 0$$
$$6P^3-6P^2-2P^2+2P-4P+4\geq 0$$
$$6P^2(P-1)-2P(P-1)-4(P-1)\geq 0$$
$$(P-1)(3P^2-P-2)\geq 0$$
$$(P-1)(3P^2-3P+2P-2)\geq 0$$
$$(P-1)[3P(P-1)+2(P-1)]\geq 0$$
$$(P-1)^2(3P+2)\geq 0 \qquad \text{(true)}$$
$$a,b>0\Rightarrow ab>0\Rightarrow P>0\Rightarrow 3P+2>0$$

Equality holds for $ab=1\Rightarrow a=b=1$ $\square$

04. If $a,b>0; ab=1$ then:

$$\frac{a}{b^3+1}+\frac{b}{a^3+1}\geq 1$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} S=a+b>0 \\ P=ab>0 \end{cases}$

(1) $$S=a+b\overset{\text{AM-GM}}{\geq} 2\sqrt{ab}=2\cdot\sqrt{1}=2\Rightarrow S\geq 2\Rightarrow S-2\geq 0$$

$$\frac{a}{b^3+1}+\frac{b}{a^3+1}\geq 1\Leftrightarrow a(a^3+1)+b(b^3+1)\geq (a^3+1)(b^3+1)$$
$$a^4+b^4+a+b\geq (ab)^3+a^3+b^3+1$$
$$(S^2-2P)^2-2P^2+S\geq 1+S^3-3SP+1$$
$$(S^2-2)^2-2+S-1-S^3+3S-1\geq 0$$
$$S^4-4S^2+4-4-S^3+4S\geq 0$$
$$S^4-S^3-4S^2+4S\geq 0, \qquad S^3-S^2-4S+4\geq 0$$

$$S^2(S-1)-4(S-1)\geq 0, \qquad S^2-4\geq 0$$
$$(S-2)(S+2)\geq 0, S-2\geq 0 \qquad \text{(True by (1))}$$

Equality holds for $a=b=1$ □

05. Solve for real numbers:

$$\sqrt{(1+x)^3}+\sqrt{(1-x)^3}-2(\sqrt{1+x}+\sqrt{1-x})=-2$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} a=\sqrt{1+x} \\ b=\sqrt{1-x} \end{cases} \Rightarrow \begin{cases} a^3+b^3-2(a+b)=-2 \\ a^2+b^2=2 \end{cases}$

Denote: $\begin{cases} S=a+b \\ P=ab \end{cases} \Rightarrow \begin{cases} S^3-3SP-2P=-2 \\ S^2-2P=2 \end{cases}$

$$S^2-2P=2\Rightarrow S^2-2=2P\Rightarrow P=\frac{S^2-2}{2}$$
$$S^3-3S\cdot\frac{S^2-2}{2}-2S=-2$$
$$2S^3-3S^3+6S-4S+4=0$$
$$-S^3+2S+4=0\Rightarrow S^3-2S-4=0$$
$$S^3-2S^2+2S^2-4S+2s-4=0$$
$$S^2(S-2)+2S(S-2)+2(S-2)=0$$
$$(S-2)(S^2+2S+2)\geq 0$$
$$S^2+2S+2=0\Rightarrow\Delta<0$$
$$S-2=0\Rightarrow S=2\Rightarrow P=1\Rightarrow a=b=1$$
$$\Rightarrow\sqrt{1+x}=\sqrt{1-x}=1\Rightarrow x=0$$

□

06. Solve for real numbers:

$$\sqrt[4]{1+x}+\sqrt[4]{1-x}=2$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} a=\sqrt[4]{1+x} \\ b=\sqrt[4]{1-x} \end{cases}; \begin{cases} S=a+b \\ P=ab \end{cases}$

$$\begin{cases} a^4+b^4=2 \\ a+b=2 \end{cases} \Rightarrow \begin{cases} (S^2-2P)^2-2P^2=2 \\ S=2 \end{cases}$$
$$(4-2P^2)^2-2P^2=2\Rightarrow 16-16P+4P^2-2P^2=2$$
$$2P^2-16P+14=0\Rightarrow P^2-8P+7=0$$
$$P=7; S=2\Rightarrow z^2-Sz+P=0\Rightarrow$$
$$\Rightarrow z^2-2z+7=0;\Delta<0$$

$$P = 1; S = 2 \Rightarrow a = b = 1 \Rightarrow 1 - x = 1 + x = 1 \Rightarrow x = 0$$

$\square$

07. Solve for real numbers:

$$\sqrt{9+x} + \sqrt{9-x} + \sqrt{81-x^2} = 15$$

Proposed by Marin Chirciu - Romania

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} a = \sqrt{9+x} \\ b = \sqrt{9-x} \end{cases} \Rightarrow \begin{cases} a^2 + b^2 = 18 \\ a + b + ab = 15 \end{cases}$

Denote: $\begin{cases} S = a + b \\ P = ab \end{cases} \Rightarrow \begin{cases} S^2 - 2P = 18 \\ S + P = 15 \end{cases}$

$$P = 15 - S \Rightarrow S^2 - 2(15 - S) = 18$$

$$S^2 - 30 + 2s - 18 = 0 \Rightarrow S^2 + 2S - 48 = 0$$

$$\Delta = 4 + 192 = 196$$

$$S = \frac{-2 + 14}{2} = \frac{12}{2} = 6; P = 15 - 6 = 9$$

$$\Rightarrow \begin{cases} a + b = 6 \\ ab = 9 \end{cases} \Rightarrow a = b = 3 \Rightarrow \begin{cases} \sqrt{9+x} = 3 \\ \sqrt{9-x} = 3 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 9 + x = 9 \\ 9 - x = 9 \end{cases} \Rightarrow x = 0$$

$\square$

08. Let be $\lambda \geq 0$ fixed. Solve for reals:

$$\sqrt{\lambda^2 + x} + \sqrt{\lambda^2 - x} + \sqrt{\lambda^4 - x^2} = \lambda^2 + 2\lambda$$

Proposed by Marin Chirciu - Romania

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases} a = \sqrt{\lambda^2 + x} \\ b = \sqrt{\lambda^2 - x} \end{cases} \Rightarrow \begin{cases} a^2 + b^2 = 2\lambda^2 \\ a + b + ab = \lambda^2 + 2\lambda \end{cases}$

Denote: $\begin{cases} S = a + b \\ P = ab \end{cases} \Rightarrow \begin{cases} S^2 - 2P = 2\lambda^2 \\ S + P = \lambda^2 + 2\lambda \end{cases}$

$$P = \lambda^2 + 2\lambda - S \Rightarrow S^2 - 2(\lambda^2 + 2\lambda - S) = 2\lambda^2$$

$$S^2 + 2S - 2\lambda^2 - 4\lambda - 2\lambda^2 = 0$$

$$S^2 + 2S - 4\lambda^2 - 4\lambda = 0$$

$$\Delta = 4 + 16\lambda^2 + 16\lambda = 4(4\lambda^2 + 4\lambda + 1) = 4(2\lambda + 1)^2$$

$$S = \frac{-2 + 2(2\lambda + 1)}{2} = \frac{-2 + 4\lambda + 2}{2} = 2\lambda$$

$$P = \lambda^2 + 2\lambda - 2\lambda = \lambda^2$$

$$S=2\lambda; P=\lambda^2 \Rightarrow a=b=\lambda \Rightarrow \begin{cases}\sqrt{\lambda^2+x}=\lambda\\ \sqrt{\lambda^2-x}=\lambda\end{cases} \Rightarrow \begin{cases}\lambda^2+x=\lambda^2\\ \lambda^2-x=\lambda^2\end{cases} \Rightarrow x=0$$

□

09. Let be $\lambda \geq 3$ fixed. Solve for reals:

$$x+\sqrt{x+\frac{5}{2}+\sqrt{x+\frac{9}{4}}}=\frac{\lambda^2-8}{4}$$

Proposed by Marin Chirciu - Romania

Solution by Daniel Sitaru - Romania.

$$x+\sqrt{x+\frac{9}{4}+2\cdot\frac{1}{2}\sqrt{x+\frac{9}{4}}+\frac{1}{4}}=\frac{\lambda^2-8}{4}$$

$$x+\sqrt{\left(\sqrt{x+\frac{9}{4}}+\frac{1}{2}\right)^2}=\frac{\lambda^2-8}{4}, \qquad x+\sqrt{x+\frac{9}{4}}+\frac{1}{2}-\frac{\lambda^2-8}{4}=0$$

Denote: $x+\frac{9}{4}=a^2 \Rightarrow x=a^2-\frac{9}{4}$

$$a^2-\frac{9}{4}+\sqrt{a^2}+\frac{1}{2}-\frac{\lambda^2}{4}+2=0$$

$$a^2+a+\frac{1}{2}+2-\frac{9}{4}-\frac{\lambda^2}{4}=0$$

$$4a^2+4a+2+8-9-\lambda^2=0$$

$$4a^2+4a+1-\lambda^2=0$$

$$\Delta=16-16(1-\lambda^2)=16\lambda^2; \sqrt{\Delta}=4\lambda$$

$$a=\frac{-4+4\lambda}{8}=\frac{\lambda-1}{2}$$

$$x=\left(\frac{\lambda-1}{2}\right)^2-\frac{9}{4}=\frac{\lambda^2-2\lambda+1}{4}-\frac{9}{4}=\frac{\lambda^2-2\lambda}{4}-2$$

□

10. Solve for real numbers:

$$\sqrt{(1+x)^3}+\sqrt{(1-x)^3}-\sqrt{1-x^2}=1$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

Denote: $\begin{cases}a=\sqrt{1+x}\\ b=\sqrt{1-x}\end{cases}$; $\begin{cases}S=a+b\\ P=ab\end{cases}$; $a,b>0 \Rightarrow S>0$

$$\begin{cases}a^3+b^3-ab=1\\ a^2+b^2=2\end{cases} \Rightarrow \begin{cases}S^3-3SP-P=1\\ S^2-2P=1\end{cases}$$

$$S^2-2=2P \Rightarrow P=\frac{S^2-2}{2}$$

$$S^3-3S\cdot\frac{S^2-2}{2}-\frac{S^2-2}{2}=1$$

$$2S^3 - 3S^3 + 6S - S^2 + 2 = 2$$
$$-S^3 - S^2 + 6S = 0 \Rightarrow -S(S^2 + S - 6) = 0$$
$$S^2 + S - 6 = 0 \Rightarrow \begin{cases} S_1 = 2 \Rightarrow P_1 = 1 \\ S_2 = -3 \Rightarrow P_2 = \frac{9-2}{2} = \frac{7}{2} \end{cases}$$
$$z^2 - S_2 z + P_2 = 0 \Rightarrow z^2 + 3z + \frac{7}{2} = 0 \Rightarrow \Delta < 0$$
$$z^2 - S_1 z + P_1 = 0 \Rightarrow z^2 - 2z + 1 = 0 \Rightarrow z_1 = z_2 = 1$$
$$\Rightarrow a = b = 1 \Rightarrow \sqrt{1+x} = \sqrt{1-x} = 1 \Rightarrow x = 0$$

□

11. In ΔABC the following relationship holds:

$$\frac{m_a m_b}{r_c^2} + \frac{m_b m_c}{r_a^2} + \frac{m_c m_a}{r_b^2} \geq 3$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{m_a m_b}{r_c^2} + \frac{m_b m_c}{r_a^2} + \frac{m_c m_a}{r_b^2} \overset{\text{AM-GM}}{\geq} 3 \cdot \sqrt[3]{\left(\frac{m_a m_b m_c}{r_a r_b r_c}\right)^2} \geq 3 \cdot \sqrt[3]{\left(\frac{r_a r_b r_c}{r_a r_b r_c}\right)^2} = 3$$

Equality holds for $a = b = c$. □

12. In ΔABC the following relationship holds:

$$abc \leq 3\sqrt{3}R^3$$

Proposed by Nguyen Hung Cuong - Vietnam

Solutioin by Daniel Sitaru - Romania.

$$abc = 4RF = 4Rrs \overset{\text{EULER}}{\leq} 4R \cdot \frac{R}{2} s = 2R^2 \cdot s \overset{\text{MITRINOVIC}}{\leq} 2R^2 \cdot \frac{3\sqrt{3}}{2} R = 3\sqrt{3}R^3$$

Equality holds for $a = b = c$. □

13. In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{a^4}{a+3c} \geq 18\sqrt{3}r^3$$

Proposed by Adgozel Adfozalov - Azerbaijan

Solution by Daniel Sitaru - Romania.

$$\frac{a^4}{a+3c} + \frac{b^3}{b+3a} + \frac{c^4}{c+3b} \overset{\text{HÖLDER}}{\geq} \frac{(a+b+c)^4}{9(a+3c+b+3a+c+3b)} =$$
$$= \frac{(2s)^4}{9 \cdot 8s} = \frac{2s^3}{9} \overset{\text{MITRINOVIC}}{\geq} = \frac{2 \cdot (3\sqrt{3}r)^3}{9} = \frac{2 \cdot 27 \cdot 3\sqrt{3}r^3}{9} = 18\sqrt{3}r^3$$

Equality holds for $a = b = c$. □

14. In ΔABC the following relationship holds:

$$\frac{a\cot\frac{A}{2}}{h_a}+\frac{b\cot\frac{B}{2}}{h_b}+\frac{c\cot\frac{C}{2}}{h_c}\geq 6$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{a\cot\frac{A}{2}}{h_a}+\frac{b\cot\frac{B}{2}}{h_b}+\frac{c\cot\frac{C}{2}}{h_c}=\sum_{cyc}\frac{a\cot\frac{A}{2}}{h_a}=\sum_{cyc}\frac{a}{\frac{2F}{a}\cdot\tan\frac{A}{2}}=\frac{1}{2F}\sum_{cyc}\frac{a^2}{\tan\frac{A}{2}}\overset{\text{AM-GM}}{\geq}$$

$$\geq\frac{1}{2F}\cdot 3\sqrt[3]{\frac{(abc)^2}{\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}}}=\frac{3}{2F}\sqrt[3]{\frac{(4Rrs)^2}{\frac{r}{s}}}=\frac{3}{2F}\cdot\sqrt[3]{\frac{16R^2r^2s^2\cdot s}{r}}=\frac{3s}{2F}\cdot 2\sqrt[3]{2R^2r}\geq$$

$$\overset{\text{EULER}}{\geq}\frac{6s}{2rs}\sqrt[3]{2\cdot(2r)^2\cdot r}=\frac{3}{r}\cdot 2r=6$$

Equality holds for $a=b=c$. □

15. In ΔABC the following relationship holds:

$$\sin A\sin B\sin C\geq\frac{3\sqrt{3}r^3}{R^3}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\sin A\sin B\sin C=\frac{a}{2R}\cdot\frac{b}{2R}\cdot\frac{c}{2R}=\frac{abc}{8R^3}\geq\frac{3\sqrt{3}r^3}{R^3}$$

$$\Leftrightarrow abc\geq 24\sqrt{3}r^3\Leftrightarrow 4Rrs\geq 24\sqrt{3}r^3\Leftrightarrow Rs\geq 6\sqrt{3}r^2 \qquad \text{(to prove)}$$

$$Rs\overset{\text{EULER}}{\geq}2r\cdot s\overset{\text{MITRINOVIC}}{\geq}2r\cdot 3\sqrt{3}r=6\sqrt{3}r^2$$

□

16. In ΔABC the following relationship holds:

$$\cos A\cos B\cos C\leq\frac{r}{4R}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\cos A\cos B\cos C=\frac{s^2-(2R+r)^2}{4R^2}\leq\frac{r}{4R}\Leftrightarrow$$

$$\Leftrightarrow s^2-(4R^2+4Rr+r^2)\leq Rr$$

$$s^2\leq 4R^2+4Rr+r^2+Rr=4R^2+5Rr+r^2$$

$$s^2\overset{\text{GERRETSEN}}{\leq}4R^2+4Rr+3r^2\leq 4R^2+5Rr+r^2\Leftrightarrow 2r^2\leq Rr\Leftrightarrow R\geq 2r \qquad \text{(Euler)}$$

Equality holds for $A=B=C$. □

17. In ΔABC the following relationship holds:

$$\frac{a}{r_b^2}+\frac{b}{r_c^2}+\frac{c}{r_a^2}\geq\frac{4\sqrt{3}}{3R}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{a}{r_b^2}+\frac{b}{r_c^2}+\frac{c}{r_a^2}\geq 3\sqrt[3]{\frac{abc}{(r_ar_br_c)^2}}=3\sqrt[3]{\frac{abc}{\left(\frac{F}{s-a}\cdot\frac{F}{s-b}\cdot\frac{F}{s-c}\right)^2}}=3\sqrt[3]{\frac{abc}{\left(\frac{F^3s}{F^2}\right)^2}}=$$

$$=3\sqrt[3]{\frac{4RF}{(Fs)^2}}=3\sqrt[3]{\frac{4RF}{F^2s^2}}=3\sqrt[3]{\frac{4R}{Fs^2}}=3\sqrt[3]{\frac{4R}{rs^3}}=\frac{3}{s}\sqrt[3]{\frac{4R}{r}}\overset{\text{EULER}}{\geq}\frac{3}{s}\sqrt[3]{\frac{8r}{r}}=$$

$$=\frac{3}{s}\cdot\sqrt[3]{8}\overset{\text{MITRINOVIC}}{\geq}\frac{3}{\frac{3\sqrt{3}R}{2}}\cdot 2=\frac{4}{\sqrt{3}R}=\frac{4\sqrt{3}}{3R}$$

Equality holds for $a=b=c$. □

18. In ΔABC the following relationship holds:

$$\sum_{cyc}m_a(\sin A-\sin B)\leq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solutoin by Daniel Sitaru - Romania.

WLOG: $a\leq b\leq c\Rightarrow\begin{cases}m_a\geq m_b\geq m_c\\ \sin A\leq\sin B\leq\sin C\end{cases}$

By rearrangements' inequality:

$$m_a\sin A+m_b\sin B+m_c\sin C\leq m_a\sin B+m_b\sin C+m_c\sin A$$

$$m_a(\sin A-\sin B)+m_b(\sin B-\sin C)+m_c(\sin C-\sin B)\leq 0$$

$$\sum_{cyc}m_a(\sin A-\sin B)\leq 0$$

Equality holds for $a=b=c$. □

19. In ΔABC the following relationship holds:

$$\sum_{cyc}h_a(m_a-m_b)\geq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

WLOG: $a\leq b\leq c\Rightarrow\begin{cases}h_a\geq h_b\geq h_c\\ m_a\geq m_b\geq m_c\end{cases}$

By rearrangements' inequality

$$h_am_a+h_bm_b+h_cm_c\geq h_am_b+h_bm_c+h_cm_a$$

$$h_a(m_a-m_b)+h_b(m_b-m_c)+h_c(m_c-m_a)\geq 0$$

$$\sum_{cyc} h_a(m_a - m_b) \geq 0$$

Equality holds for $a = b = c$.
Observation 1

$$a \leq b \Leftrightarrow \frac{1}{a} \geq \frac{1}{b} \Leftrightarrow \frac{2F}{a} \geq \frac{2F}{b} \Leftrightarrow h_a \geq h_b$$

Observation 2

$$a \leq b \Leftrightarrow m_a \geq m_b$$

$$m_a \geq m_b \Leftrightarrow m_a^2 \geq m_b^2 \Leftrightarrow \frac{1}{2}(b^2 + c^2) - \frac{1}{4}a^2 \geq \frac{1}{2}(a^2 + c^2) - \frac{1}{4}b^2$$

$$\Leftrightarrow 2(b^2 + c^2) - a^2 \geq 2(a^2 + b^2) - b^2$$

$$\Leftrightarrow 2b^2 + 2c^2 - a^2 \geq 2a^2 + 2c^2 - b^2 \Leftrightarrow 3a^2 \leq 3b^2 \Leftrightarrow a^2 \leq b^2 \Leftrightarrow a \leq b$$

□

20. In ΔABC the following relationship holds:

$$\frac{GA}{r_b} + \frac{GB}{r_c} + \frac{GC}{r_a} \geq 2$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{GA}{r_b} + \frac{GB}{r_c} + \frac{GC}{r_a} \geq 3\sqrt[3]{\frac{GA \cdot GB \cdot GC}{r_a r_b r_c}} =$$

$$= 3\sqrt[3]{\frac{\frac{2}{3}m_a \cdot \frac{2}{3}m_b \cdot \frac{2}{3}m_c}{r_a r_b r_c}} = 3 \cdot \frac{2}{3}\sqrt[3]{\frac{m_a m_b m_c}{r_a r_b r_c}} \geq 2\sqrt[3]{\frac{r_a r_b r_c}{r_a r_b r_c}} = 2$$

Equality holds for $a = b = c$. □

21. In ΔABC the following relationship holds:

$$\frac{GA \cdot GB}{r_c^2} + \frac{GB \cdot GC}{r_a^2} + \frac{GC \cdot GA}{r_b^2} \geq \frac{4}{3}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{GA \cdot GB}{r_c^2} + \frac{GB \cdot GC}{r_a^2} + \frac{GC \cdot GA}{r_b^2} \geq 3 \cdot \sqrt[3]{\left(\frac{GA \cdot GB \cdot GC}{r_a r_b r_c}\right)^2} = 3\sqrt[3]{\left(\frac{\frac{2}{3}m_a \cdot \frac{2}{3}m_b \cdot \frac{2}{3}m_c}{r_a r_b r_c}\right)^2} =$$

$$= 3 \cdot \left(\frac{2}{3}\right)^2 \cdot \sqrt[3]{\left(\frac{m_a m_b m_c}{r_a r_b r_c}\right)^2} = 3 \cdot \frac{4}{9} \cdot \sqrt[3]{\left(\frac{r_a r_b r_c}{r_a r_b r_c}\right)^2} = \frac{4}{3}$$

Equality holds for $a = b = c$. □

22. In acute ΔABC the following relationship holds:

$$\cot\frac{A}{2}\csc A + \cot\frac{B}{2}\csc B + \cot\frac{C}{2}\csc C \geq 6$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\cot\frac{A}{2}\csc A+\cot\frac{B}{2}\csc B+\cot\frac{C}{2}\csc C\overset{\text{AM-GM}}{\geq}3\cdot\sqrt[3]{\frac{\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}}{\sin A\sin B\sin C}}=$$

$$=3\cdot\sqrt[3]{\frac{\frac{s}{r}}{\frac{a}{2R}\cdot\frac{b}{2R}\cdot\frac{c}{2R}}}=3\cdot 2R\cdot\sqrt[3]{\frac{s}{abcr}}=6R\cdot\sqrt[3]{\frac{s}{4Rrsr}}=6R\sqrt[3]{\frac{1}{4Rr^2}}\overset{\text{EULER}}{\geq}$$

$$\geq 6R\cdot\sqrt[3]{\frac{1}{4R\cdot\frac{R^2}{4}}}=6R\cdot\frac{1}{R}=6$$

Equality holds for $a=b=c$. $\square$

23. In ΔABC the following relationship holds:

$$\sum_{cyc}a(h_a-h_b)\leq 0$$

Proposed by Nguyen Hung Cuong - -Vietnam

Solution by Daniel Sitaru - Romania.
WLOG: $a\leq b\leq c\Rightarrow h_a\geq h_b\geq h_c$.
By rearrangements' inequality:

$$ah_a+bh_b+ch_c\leq ah_b+bh_c+ch_a$$
$$a(h_a-h_b)+b(h_b-h_c)+c(h_c-h_a)\leq 0$$
$$\sum_{cyc}a(h_a-h_b)\leq 0$$

Equality holds for $a=b=c$.
Observation:

$$a\leq b\Rightarrow h_a\geq h_b$$
$$h_a\geq h_b\Leftrightarrow\frac{2F}{a}\geq\frac{2F}{b}\Leftrightarrow\frac{1}{a}\geq\frac{1}{b}\Leftrightarrow a\leq b$$

$\square$

24. In ΔABC the following relationship holds:

$$\sum_{cyc}a(\cos A-\cos B)\leq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.
WLOG: $a\leq b\leq c\Rightarrow\cos A\geq\cos B\geq\cos C$
By rearrangements' inequality:

$$a\cos A+b\cos B+\cos C\leq a\cos B+b\cos C+\cos A$$
$$a(\cos A-\cos B)+b(\cos B-\cos C)+c(\cos C-\cos A)\leq 0$$
$$\sum_{cyc}a(\cos A-\cos B)\leq 0$$

Equality holds for $a=b=c$. $\square$

25. In ΔABC the following relationship holds:

$$\frac{a^2}{w_bw_c}+\frac{b^2}{w_cw_a}+\frac{c^2}{w_aw_b}\geq 4$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{a^2}{w_bw_c}+\frac{b^2}{w_cw_a}+\frac{c^2}{w_aw_b}\geq 3\sqrt[3]{\left(\frac{abc}{w_aw_bw_c}\right)^2}\geq 3\sqrt[3]{\left(\frac{abc}{\sqrt{s(s-a)}\cdot\sqrt{s(s-b)}\cdot\sqrt{s(s-c)}}\right)^2}=$$

$$=3\sqrt[3]{\left(\frac{abc}{sF}\right)^2}=3\sqrt[3]{\left(\frac{4RF}{sF}\right)^2}=3\sqrt[3]{\left(\frac{4R}{s}\right)^2}\overset{\text{MITRINOVIC}}{\geq}3\sqrt[3]{\left(\frac{4R}{\frac{3\sqrt{3}}{2}R}\right)^2}=3\sqrt[3]{\left(\frac{8}{3\sqrt{3}}\right)^2}=$$

$$=3\sqrt[3]{\frac{64}{27}}=3\cdot\frac{4}{3}=4$$

Equality holds for $a=b=c$. □

26. In ΔABC the following relationship holds:

$$\sum_{cyc}a(w_a-w_b)\leq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.
WLOG: $a\leq b\leq c\Rightarrow w_a\geq w_b\geq w_c$
By rearrangements' inequality:

$$aw_a+bw_b+cw_c\leq aw_b+bw_c+cw_a$$

$$a(w_a-w_b)+b(w_b-w_c)+c(w_c-w_a)\leq 0$$

$$\sum_{cyc}a(w_a-w_b)\leq 0$$

Equality holds for $a=b=c$. □

27. In ΔABC the following relationship holds:

$$\frac{r_a^2}{h_bh_c}+\frac{r_b^2}{h_ch_a}+\frac{r_c^2}{h_ah_b}\geq 3$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{r_a^2}{h_bh_c}+\frac{r_b^2}{h_ch_a}+\frac{r_c^2}{h_ah_b}\geq 3\sqrt[3]{\left(\frac{r_ar_br_c}{h_ah_bh_c}\right)^2}=3\sqrt[3]{\left(\frac{\frac{F}{s-a}\cdot\frac{F}{s-b}\cdot\frac{F}{s-c}}{\frac{2F}{a}\cdot\frac{2F}{b}\cdot\frac{2F}{c}}\right)^2}=$$

$$=\frac{3}{4}\sqrt[3]{\left(\frac{abc}{(s-a)(s-b)(s-c)}\right)^2}=\frac{3}{4}\sqrt[3]{\left(\frac{abcs}{s(s-a)(s-b)(s-c)}\right)^2}=\frac{3}{4}\sqrt[3]{\left(\frac{4RFs}{F^2}\right)^2}=$$

$$= \frac{3}{4}\sqrt[3]{\left(\frac{4Rs}{rs}\right)^2} = \frac{3}{4}\sqrt[3]{\left(\frac{4R}{r}\right)^2} \overset{\text{EULER}}{\geq} \frac{3}{4}\sqrt[3]{\left(\frac{8r}{r}\right)^2} = \frac{3}{4}\sqrt[3]{64} = \frac{3}{4}\cdot 4 = 3$$

Equality holds for $a = b = c$. □

28. In ΔABC the following relationship holds:

$$\frac{a^2}{r_a r_b} + \frac{b^2}{r_b r_c} + \frac{c^2}{r_c r_a} \geq 4$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{a^2}{r_a r_b}+\frac{b^2}{r_b r_c}+\frac{c^2}{r_c r_a} \geq 3\sqrt[3]{\left(\frac{abc}{r_a r_b r_c}\right)^2} = 3\sqrt[3]{\left(\frac{4RF}{\frac{F}{s-a}\cdot\frac{F}{s-b}\cdot\frac{F}{s-c}}\right)^2} = 3\sqrt[3]{\left(\frac{4RF}{\frac{F^3\cdot s}{s(s-a)(s-b)(s-c)}}\right)^2} =$$

$$= 3\sqrt[3]{\left(\frac{4RF\cdot F^2}{F^2 s}\right)^2} = 3\sqrt[3]{\left(\frac{4R}{2}\right)^2} \overset{\text{MITRINOVIC}}{\geq} 3\sqrt[3]{\left(\frac{3\cdot\frac{2}{3\sqrt{3}}s}{s}\right)^2} = 3\sqrt[3]{\left(\frac{8}{3\sqrt{3}}\right)^2} =$$

$$= 3\sqrt[3]{\frac{64}{27}} = 3\cdot\frac{4}{3} = 4$$

Equality holds for $a = b = c$. □

29. In ΔABC the following relationship holds:

$$\frac{a}{w_b} + \frac{b}{w_c} + \frac{c}{w_c} \geq 2\sqrt{3}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{a}{w_b} + \frac{b}{w_c} + \frac{c}{w_a} \overset{\text{AM-GM}}{\geq} 3\cdot\sqrt[3]{\frac{abc}{w_a w_b w_c}} \geq \sqrt[3]{\frac{4RF}{\sqrt{s(s-a)}\cdot\sqrt{s(s-b)}\cdot\sqrt{s(s-c)}}} =$$

$$= 3\sqrt[3]{\frac{4RF}{s\cdot F}} = 3\sqrt[3]{\frac{4R}{s}} \overset{\text{MITRINOVIC}}{\geq} 3\sqrt[3]{\frac{4\cdot\frac{2}{3\sqrt{3}}s}{s}} = 3\sqrt[3]{\frac{8}{3\sqrt{3}}} = 3\sqrt[3]{\left(\frac{2}{\sqrt{3}}\right)^2} =$$

$$= 3\cdot\frac{2}{\sqrt{3}} = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Equality holds for $a = b = c$. □

30. In ΔABC the following relationship holds:

$$\sum_{cyc} a\left(\cos\frac{A}{2} - \cos\frac{B}{2}\right) \leq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.
WLOG: $a \leq b \leq c \Rightarrow \cos\frac{A}{2} \geq \cos\frac{B}{2} \geq \cos\frac{C}{2}$
By rearrangements' inequality:

$$a\cos\frac{A}{2} + b\cos\frac{B}{2} + c\cos\frac{C}{2} \leq a\cos\frac{B}{2} + b\cos\frac{C}{2} + c\cos\frac{A}{2}$$

$$a\left(\cos\frac{A}{2} - \cos\frac{B}{2}\right) + b\left(\cos\frac{B}{2} - \cos\frac{C}{2}\right) + c\left(\cos\frac{C}{2} - \cos\frac{A}{2}\right) \leq 0$$

$$\sum_{cyc} a\left(\cos\frac{A}{2} - \cos\frac{B}{2}\right) \leq 0$$

Equality holds for $a = b = c$. □

31. In ΔABC the following relationship holds:

$$\sum_{cyc} a\left(\tan\frac{A}{2} - \tan\frac{B}{2}\right) \geq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.
WLOG: $a \leq b \leq c \Rightarrow \tan\frac{A}{2} \leq \tan\frac{B}{2} \leq \tan\frac{C}{2}$
By rearrangements' inequality:

$$a\tan\frac{A}{2} + b\tan\frac{B}{2} + c\tan\frac{C}{2} \geq 2\tan\frac{B}{2} + b\tan\frac{C}{2} + c\tan\frac{A}{2}$$

$$a\left(\tan\frac{A}{2} - \tan\frac{B}{2}\right) + b\left(\tan\frac{B}{2} - \tan\frac{C}{2}\right) + c\left(\tan\frac{C}{2} - \tan\frac{A}{2}\right) \geq 0$$

$$\sum_{cyc} a\left(\tan\frac{A}{2} - \tan\frac{B}{2}\right) \geq 0$$

Equality holds for $a = b = c$ □

32. In ΔABC the following relationship holds:

$$\frac{a}{h_a \sin^2\frac{A}{2}} + \frac{b}{h_b \sin^2\frac{B}{2}} + \frac{c}{h_c \sin^2\frac{C}{2}} \geq 8\sqrt{3}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\frac{a}{h_a \sin^2\frac{A}{2}} + \frac{b}{h_b \sin^2\frac{B}{2}} + \frac{c}{h_c \sin^2\frac{C}{2}} = \sum_{cyc} \frac{a}{h_a \cdot \sin^2\frac{A}{2}} =$$

$$= \sum_{cyc} \frac{a}{\frac{2F}{a} \cdot \frac{(s-b)(s-c)}{bc}} = \frac{abc}{2F} \sum_{cyc} \frac{a}{(s-b)(s-c)} =$$

$$= \frac{4RF}{2F} \cdot \frac{1}{(s-a)(s-b)(s-c)} \sum_{cyc} a(s-a) =$$

$$= \frac{2Rs}{s(s-a)(s-b)(s-c)} \cdot 2r(4R+r) \overset{\text{EULER}}{\geq} \frac{2Rs}{F^2} \cdot 2r(8r+r) = \frac{4Rrs \cdot 9r}{F \cdot rs} =$$

$$=\frac{36Rr}{F}=\frac{36Rr}{rs}=\frac{36R}{s}\overset{\text{MITRINOVIC}}{\geq}\frac{36R}{\frac{3\sqrt{3}}{2}R}=\frac{72}{3\sqrt{3}}=\frac{24}{\sqrt{3}}=\frac{24\sqrt{3}}{3}=8\sqrt{3}$$

Equality holds for $a=b=c$. □

33. In ΔABC the following relationship holds:

$$\sum_{cyc} h_a(w_a-w_b)\geq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.
WLOG: $a\leq b\leq c\Rightarrow \begin{cases} h_a\geq h_b\geq h_c \\ w_a\geq w_b\geq w_c \end{cases}$
By rearrangements' inequality:

$$h_aw_a+h_bw_b+h_cw_c\geq h_aw_b+h_bw_c+h_cw_a$$

$$h_a(w_a-w_b)+h_b(w_b-w_c)+h_c(w_c-w_a)\geq 0$$

$$\sum_{cyc} h_a(w_a-w_b)\geq 0$$

Equality holds for $a=b=c$. □

34. In ΔABC the following relationship holds:

$$\frac{a}{h_a\sin^2 A}+\frac{b}{h_b\sin^2 B}+\frac{c}{h_c\sin^2 C}\geq\frac{8\sqrt{3}}{3}$$

Proposed by Nguyen Hung Cuong - Vietnam

Soltuion by Daniel Sitaru - Romania.

$$\frac{a}{h_a\sin^2 A}+\frac{b}{h_b\sin^2 B}+\frac{c}{h_c\sin^2 C}=\sum_{cyc}\frac{a}{h_a\sin^2 A}=\sum\frac{a}{\frac{2F}{a}\cdot\left(\frac{a}{2r}\right)^2}=\sum\frac{a\cdot 4R^2}{2F\cdot a}=$$

$$=\frac{4R^2}{2F}\cdot 3=\frac{12R^2}{2F}=\frac{6R^2}{rs}\overset{\text{EULER}}{\geq}\frac{6R\cdot 2r}{rs}=\frac{12R}{s}\overset{\text{MITRINOVIC}}{\geq}\frac{12R}{\frac{3\sqrt{3}R}{2}}=\frac{24}{3\sqrt{3}}=$$

$$=\frac{8}{\sqrt{3}}=\frac{8\sqrt{3}}{3}$$

Equality holds for $a=b=c$. □

35. In ΔABC the following relationship holds:

$$\sum_{cyc} a(\sin A-\sin B)\geq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution 1 by Daniel Sitaru - Romania.

$$\sum_{cyc} a(\sin A - \sin B) \geq 0 \Leftrightarrow \sum_{cyc} a\left(\frac{a}{2R} - \frac{b}{2R}\right) \geq 0$$

$$\Leftrightarrow \sum_{cyc} a(a-b) \geq 0 \Leftrightarrow \sum_{cyc} a^2 \geq \sum_{cyc} ab$$

$$\Leftrightarrow a^2 + b^2 + c^2 \geq ab + bc + ca \Leftrightarrow$$

$$\frac{1}{2}(a-b)^2 + \frac{1}{2}(b-c)^2 + \frac{1}{2}(c-a)^2 \geq 0 \qquad \text{(True!)}$$

□

Solution 2 by Daniel Sitaru - Romania.
WLOG: $a \leq b \leq c \Rightarrow \sin A \leq \sin B \leq \sin C$
By rearrangements' inequality:

$$a \sin A + b \sin B + c \sin C \geq a \sin B + b \sin C + c \sin A$$

$$a(\sin A - \sin B) + b(\sin B - \sin C) + c(\sin C - \sin B) \geq 0$$

$$\sum_{cyc} a(\sin A - \sin B) \geq 0$$

Equality holds for $a = b = c$. □

36. In ΔABC the following relationship holds:

$$\sum_{cyc} a\left(\cot\frac{A}{2} - \cot\frac{B}{2}\right) \leq 0$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.
WLOG: $a \leq b \leq c \Rightarrow \cot\frac{A}{2} \geq \cot\frac{B}{2} \geq \frac{C}{2}$
By rearrangements' inequality:

$$a \cot\frac{A}{2} + b \cot\frac{B}{2} + c \cot\frac{C}{2} \leq a \cot\frac{B}{2} + b \cot\frac{C}{2} + \cot\frac{A}{2}$$

$$a\left(a \cot\frac{A}{2} - \cot\frac{B}{2}\right) + b\left(\cot\frac{B}{2} - \cot\frac{C}{2}\right) + c\left(\cot\frac{C}{2} - \cot\frac{A}{2}\right) \leq 0$$

$$\sum_{cyc} a\left(\cot\frac{A}{2} - \cot\frac{B}{2}\right) \leq 0$$

Equality holds for $a = b = c$. □

37. Find:

$$\Omega = \int_0^\infty \frac{dx}{x\sqrt{x}+1}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution 1 by Daniel Sitaru - Romania.

$$\Omega = \int_0^\infty \frac{dx}{x\sqrt{x}+1} \overset{(x=y^2)}{=} \int_0^\infty \frac{2ydy}{y^3+1}$$

$$\frac{2y}{(y+1)(y^2-y+1)} = \frac{A}{y+1} + \frac{By+C}{y^2-y+1}$$

$$2y = A(y^2-y+1) + (y+1)(By+C)$$

$$2y = Ay^2 - Ay + A + By^2 + Cy + By + C$$

$$2y = (A+B)y^2 + y(-A+B+C) + A + C$$

$$\begin{cases} A+B=0 \\ -A+B+C=2 \\ A+C \end{cases} \Rightarrow B=-A \Rightarrow \begin{cases} -2A+C=2 \\ 2A+C=0 \end{cases}$$

$$\Rightarrow 3C = 2 \Rightarrow C = \frac{2}{3} \Rightarrow A = -\frac{2}{3} \Rightarrow B = \frac{2}{3}$$

$$\Omega = \int_0^\infty \left(\frac{-\frac{2}{3}}{y+1} + \frac{\frac{2}{3}(y+1)}{y^2-y+1} \right) dy =$$

$$= -\frac{2}{3}\ln(y+1)\Big|_0^\infty + \frac{1}{3}\int_0^\infty \frac{2y-1+3}{y^2-y+1} dy =$$

$$= -\frac{2}{3}\ln(y+1) + \frac{1}{3}\ln(y^2-y+1)\Big|_0^\infty + \int_0^\infty \frac{1}{\left(y-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{3}\ln\left(\frac{y^2-y+1}{y^2+2y+1}\right)\Big|_0^\infty + \frac{1}{\frac{\sqrt{3}}{2}}\arctan\left(\frac{y-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\Bigg|_0^\infty =$$

$$= \frac{1}{3}(\ln 1 - \ln 1) + \frac{2}{\sqrt{3}}\left(\arctan\infty - \arctan\left(-\frac{1}{\sqrt{3}}\right)\right) =$$

$$= \frac{1}{3}(0-0) + \frac{2}{\sqrt{3}}\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \cdot \frac{4\pi}{6} = \frac{8\pi}{6\sqrt{3}} = \frac{4\pi}{3\sqrt{3}}$$

□

Solution 2 by Daniel Sitaru - Romania.

$$\int_0^\infty \frac{dx}{x^p+1} = \frac{\pi}{p\sin\left(\frac{\pi}{p}\right)}$$

$$\Omega = \int_0^\infty \frac{dx}{x^{\frac{2}{3}}+1} = \frac{\pi}{\frac{3}{2}\sin\left(\frac{\pi}{\frac{3}{2}}\right)} = \frac{2\pi}{3\sin\frac{2\pi}{3}} = \frac{2\pi}{3\sin\left(\pi-\frac{\pi}{3}\right)} = \frac{2\pi}{3\cdot\sin\frac{\pi}{3}} = \frac{2\pi}{\frac{3\sqrt{3}}{2}} = \frac{4\pi}{3\sqrt{3}}$$

□

38. Find:

$$\Omega = \int_0^\infty \frac{dx}{x\sqrt{x}+x+\sqrt{x}+1}$$

Proposed by Nguyen Hung Cuong - Vietnam

Solution by Daniel Sitaru - Romania.

$$\Omega = \int_0^\infty \frac{dx}{x\sqrt{x}+x+\sqrt{x}+1} = \int_0^\infty \frac{dx}{(\sqrt{x}+1)(x+1)} \overset{(x=y^2)}{=}$$

$$= \int_0^\infty \frac{2ydy}{(y+1)(y^2+1)} = \int_0^\infty \Big(\frac{-1}{y+1} + \frac{y}{y^2+1} + \frac{1}{y^2+1}\Big)dy =$$

$$= \Big(-\ln(y+1) + \frac{1}{2}\ln(y^2+1)\Big)\Big|_0^\infty + \arctan y\Big|_0^\infty =$$

$$= \Big(\ln\frac{\sqrt{y^2+1}}{y+1}\Big)\Big|_0^\infty + \arctan\infty - \arctan 0 = \ln 1 - \ln 1 + \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

□

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