

ROMANIAN MATHEMATICAL MAGAZINE

In acute $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} h_a (\tan A - \tan B) \leq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} h_a \geq h_b \geq h_c \\ \tan A \leq \tan B \leq \tan C \end{cases}$$

By rearrangements' inequality:

$$h_a \tan A + h_b \tan B + h_c \tan C \leq h_a \tan B + h_b \tan C + h_c \tan A$$

$$h_a (\tan A - \tan B) + h_b (\tan B - \tan C) + h_c (\tan C - \tan A) \leq 0$$

$$\sum_{cyc} h_a (\tan A - \tan B) \leq 0$$

Equality holds for $a = b = c$.