

ROMANIAN MATHEMATICAL MAGAZINE

In acute $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} h_a(\cot A - \cot B) \geq 0$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} h_a \geq h_b \geq h_c \\ \cot A \geq \cot B \geq \cot C \end{cases}$$

By rearrangements' inequality:

$$h_a \cot A + h_b \cot B + h_c \cot C \geq h_a \cot B + h_b \cot C + h_c \cot A$$

$$h_a(\cot A - \cot B) + h_b(\cot B - \cot C) + h_c(\cot C - \cot A) \geq 0$$

$$\sum_{cyc} h_a(\cot A - \cot B) \geq 0$$

Equality holds for $a = b = c$.