

ROMANIAN MATHEMATICAL MAGAZINE

In acute $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} w_a (\tan A - \tan B) \leq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} w_a \geq w_b \geq w_c \\ \tan A \leq \tan B \leq \tan C \end{cases}$$

By rearrangements' inequality:

$$w_a \tan A + w_b \tan B + w_c \tan C \leq w_a \tan B + w_b \tan C + w_c \tan A$$

$$w_a (\tan A - \tan B) + w_b (\tan B - \tan C) + w_c (\tan C - \tan A) \leq 0$$

$$\sum_{cyc} w_a (\tan A - \tan B) \leq 0$$

Equality holds for $a = b = c$.