

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} r_a (\sin A - \sin B) \geq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow -a \geq -b \geq -c \Rightarrow s - a \geq s - b \geq s - c$$

$$\Rightarrow \frac{1}{s-a} \leq \frac{1}{s-b} \leq \frac{1}{s-c} \Rightarrow \frac{F}{s-a} \leq \frac{F}{s-b} \leq \frac{F}{s-c} \Rightarrow r_a \leq r_b \leq r_c$$

$$a \leq b \leq c \Rightarrow \sin A \leq \sin B \leq \sin C$$

By rearrangements' inequality:

$$r_a \sin A + r_b \sin B + r_c \sin C \leq r_a \sin B + r_b \sin C + r_c \sin A$$

$$r_a (\sin A - \sin B) + r_b (\sin B - \sin C) + r_c (\sin C - \sin A) \leq 0$$

$$\sum_{cyc} r_a (\sin A - \sin B) \leq 0$$

Equality holds for $a = b = c$.