

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} h_a(w_a - w_b) \geq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} h_a \geq h_b \geq h_c \\ w_a \geq w_b \geq w_c \end{cases}$$

By rearrangements' inequality:

$$\begin{aligned} h_a w_a + h_b w_b + h_c w_c &\leq h_a w_b + h_b w_c + h_c w_a \\ h_a(w_a - w_b) + h_b(w_b - w_c) + h_c(w_c - w_a) &\leq 0 \end{aligned}$$

$$\sum_{cyc} h_a(w_a - w_b) \geq 0$$

Equality holds for $a = b = c$.