

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} w_a \left(\cot \frac{A}{2} - \cot \frac{B}{2} \right) \geq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} w_a \geq w_b \geq w_c \\ \cot \frac{A}{2} \geq \cot \frac{B}{2} \geq \cot \frac{C}{2} \end{cases}$$

By rearrangements' inequality:

$$w_a \cot \frac{A}{2} + w_b \cot \frac{B}{2} + w_c \cot \frac{C}{2} \geq w_a \cot \frac{B}{2} + w_b \cot \frac{C}{2} + w_c \cot \frac{A}{2}$$

$$w_a \left(\cot \frac{A}{2} - \cot \frac{B}{2} \right) + w_b \left(\cot \frac{B}{2} - \cot \frac{C}{2} \right) + w_c \left(\cot \frac{C}{2} - \cot \frac{A}{2} \right) \geq 0$$

$$\sum_{cyc} w_a \left(\cot \frac{A}{2} - \cot \frac{B}{2} \right) \geq 0$$

Equality holds for $a = b = c$.