

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\sum_{cyc} w_a \left( \sin \frac{A}{2} - \sin \frac{B}{2} \right) \leq 0$$

*Proposed by Nguyen Hung Cuong – Vietnam*

*Solution by Daniel Sitaru – Romania*

$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} w_a \geq w_b \geq w_c \\ \sin \frac{A}{2} \leq \sin \frac{B}{2} \leq \sin \frac{C}{2} \end{cases}$$

By rearrangements' inequality:

$$\begin{aligned} w_a \sin \frac{A}{2} + w_b \sin \frac{B}{2} + w_c \sin \frac{C}{2} &\leq w_a \sin \frac{B}{2} + w_b \sin \frac{C}{2} + w_c \sin \frac{A}{2} \\ w_a \left( \sin \frac{A}{2} - \sin \frac{B}{2} \right) + w_b \left( \sin \frac{B}{2} - \sin \frac{C}{2} \right) + w_c \left( \sin \frac{C}{2} - \sin \frac{A}{2} \right) &\leq 0 \\ \sum_{cyc} w_a \left( \sin \frac{A}{2} - \sin \frac{B}{2} \right) &\leq 0 \end{aligned}$$

Equality holds for  $a = b = c$ .