

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\sum_{cyc} h_a(r_a - r_b) \leq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \frac{1}{a} \geq \frac{1}{b} \geq \frac{1}{c} \Rightarrow \frac{2F}{a} \geq \frac{2F}{b} \geq \frac{2F}{c} \Rightarrow h_a \geq h_b \geq h_c$$

$$a \leq b \leq c \Rightarrow -a \geq -b \geq -c \Rightarrow s - a \geq s - b \geq s - c$$

$$\Rightarrow \frac{1}{s-a} \leq \frac{1}{s-b} \leq \frac{1}{s-c} \Rightarrow \frac{F}{s-a} \leq \frac{F}{s-b} \leq \frac{F}{s-c} \Rightarrow r_a \leq r_b \leq r_c$$

By rearrangements' inequality:

$$h_a r_a + h_b r_b + h_c r_c \leq h_a r_b + h_b r_c + h_c r_a$$

$$h_a(r_a - r_b) + h_b(r_b - r_c) + h_c(r_c - r_a) \leq 0$$

$$\sum_{cyc} h_a(r_a - r_b) \leq 0$$

Equality holds for  $a = b = c$ .