

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} m_a \left(\cos \frac{A}{2} - \cos \frac{B}{2} \right) \geq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} m_a \geq m_b \geq m_c \\ \cos \frac{A}{2} \geq \cos \frac{B}{2} \geq \cos \frac{C}{2} \end{cases}$$

By rearrangements' inequality:

$$m_a \cos \frac{A}{2} + m_b \cos \frac{B}{2} + m_c \cos \frac{C}{2} \geq m_a \cos \frac{B}{2} + m_b \cos \frac{C}{2} + m_c \cos \frac{A}{2}$$

$$m_a \left(\cos \frac{A}{2} - \cos \frac{B}{2} \right) + m_b \left(\cos \frac{B}{2} - \cos \frac{C}{2} \right) + m_c \left(\cos \frac{C}{2} - \cos \frac{A}{2} \right) \geq 0$$

Equality holds for $a = b = c$.