

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\sum_{cyc} \sin\left(\frac{A}{2}\right) \cos(A) \leq \frac{3}{4}$$

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$$\sum_{cyc} \sin\left(\frac{A}{2}\right) \cos(A) = \sum_{cyc} \sin\left(\frac{A}{2}\right) \left(1 - 2\sin^2\left(\frac{A}{2}\right)\right) = \sum_{cyc} \left(\sin\left(\frac{A}{2}\right) - 2\sin^3\left(\frac{A}{2}\right)\right)$$

$$\text{Let } \sin\left(\frac{A}{2}\right) = x, \quad x \in (0; 1), \quad f(x) = x - 2x^3 \rightarrow f''(x) = -12x < 0$$

*So  $f(x)$  is concave function : By Jensen's Inequality*

$$\begin{aligned} \sum_{cyc} \left(\sin\left(\frac{A}{2}\right) - 2\sin^3\left(\frac{A}{2}\right)\right) &\leq 3 \left( \sin \frac{\frac{A}{2} + \frac{B}{2} + \frac{C}{2}}{3} - 2\sin^3 \frac{\frac{A}{2} + \frac{B}{2} + \frac{C}{2}}{3} \right) = \\ &= 3 \left( \frac{1}{2} - 2\left(\frac{1}{2}\right)^3 \right) = \frac{3}{4} \end{aligned}$$

*Equality holds iff  $a = b = c$*