

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} a(r_a - r_b) \leq 0$$

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$$\sum_{cyc} a(m_a - m_b) \leq 0, \quad ar_a + br_b + cr_c \geq ar_b + br_c + cr_a (*)$$

Let us prove that :

$$\text{Wlog : } a \leq b \leq c \rightarrow r_a \leq r_b \leq r_c$$

These conditions are suitable for Rearrangement inequality:

$$A_{max} = ar_a + br_b + cr_c, \quad C_{mixed} = ar_b + br_c + cr_a$$

$$\text{So : } A_{max} \geq C_{mixed} \text{ (Proved)}$$

Equality holds iff $a = b = c$