

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} a (\sin A - \sin B) \geq 0$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution 1 by Daniel Sitaru – Romania

$$\begin{aligned} \sum_{cyc} a (\sin A - \sin B) \geq 0 &\Leftrightarrow \sum_{cyc} a \left(\frac{a}{2R} - \frac{b}{2R} \right) \geq 0 \\ &\Leftrightarrow \sum_{cyc} a(a - b) \geq 0 \Leftrightarrow \sum_{cyc} a^2 \geq \sum_{cyc} ab \\ &\Leftrightarrow a^2 + b^2 + c^2 \geq ab + bc + ca \Leftrightarrow \\ &\Leftrightarrow \frac{1}{2}(a - b)^2 + \frac{1}{2}(b - c)^2 + \frac{1}{2}(c - a)^2 \geq 0 \quad (\text{True!}) \end{aligned}$$

Equality holds for $a = b = c$.

Solution 2 by Daniel Sitaru – Romania

$$\text{WLOG: } a \leq b \leq c \Rightarrow \sin A \leq \sin B \leq \sin C$$

By rearrangements' inequality:

$$\begin{aligned} a \sin A + b \sin B + c \sin C &\geq a \sin B + b \sin C + c \sin A \\ a(\sin A - \sin B) + b(\sin B - \sin C) + c(\sin C - \sin A) &\geq 0 \end{aligned}$$

$$\sum_{cyc} a(\sin A - \sin B) \geq 0$$

Equality holds for $a = b = c$.