

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} a \left(\cos \frac{A}{2} - \cos \frac{B}{2} \right) \leq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \cos \frac{A}{2} \geq \cos \frac{B}{2} \geq \cos \frac{C}{2}$$

By rearrangements' inequality:

$$\begin{aligned} a \cos \frac{A}{2} + b \cos \frac{B}{2} + c \cos \frac{C}{2} &\leq a \cos \frac{B}{2} + b \cos \frac{C}{2} + c \cos \frac{A}{2} \\ a \left(\cos \frac{A}{2} - \cos \frac{B}{2} \right) + b \left(\cos \frac{B}{2} - \cos \frac{C}{2} \right) + c \left(\cos \frac{C}{2} - \cos \frac{A}{2} \right) &\leq 0 \\ \sum_{cyc} a \left(\cos \frac{A}{2} - \cos \frac{B}{2} \right) &\leq 0 \end{aligned}$$

Equality holds for $a = b = c$.