

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} a(w_a - w_b) \leq 0$$

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WLOG: $a \leq b \leq c \Rightarrow w_a \geq w_b \geq w_c$

By rearrangements' inequality:

$$\begin{aligned}aw_a + bw_b + cw_c &\leq aw_b + bw_c + cw_a \\a(w_a - w_b) + b(w_b - w_c) + c(w_c - w_a) &\leq 0 \\ \sum_{cyc} a(w_a - w_b) &\leq 0\end{aligned}$$

Equality holds for $a = b = c$.