

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} m_a (\sin A - \sin B) \leq 0$$

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$$\text{WLOG: } a \leq b \leq c \Rightarrow \begin{cases} m_a \geq m_b \geq m_c \\ \sin A \leq \sin B \leq \sin C \end{cases}$$

By rearrangements' inequality:

$$m_a \sin A + m_b \sin B + m_c \sin C \leq m_a \sin B + m_b \sin C + m_c \sin A$$

$$m_a (\sin A - \sin B) + m_b (\sin B - \sin C) + m_c (\sin C - \sin B) \leq 0$$

$$\sum_{cyc} m_a (\sin A - \sin B) \leq 0$$

Equality holds for $a = b = c$.