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In acute ΔABC the following relationship holds:

$$\cot\frac{A}{2}\csc A+\cot\frac{B}{2}\csc B+\cot\frac{C}{2}\csc C\ge 6$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\cot\frac{A}{2}\csc A+\cot\frac{B}{2}\csc B+\cot\frac{C}{2}\csc C\overset{AM-GM}{\ge}3\cdot\sqrt[3]{\frac{\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}}{\sin A\sin B\sin C}}$$

$$=3\cdot\sqrt[3]{\frac{\frac{s}{r}}{\frac{a}{2R}\cdot\frac{b}{2R}\cdot\frac{c}{2R}}}=3\cdot 2R\cdot\sqrt[3]{\frac{s}{abcr}}=6R\cdot\sqrt[3]{\frac{s}{4Rrsr}}=6R\sqrt[3]{\frac{1}{4Rr^2}}\overset{EULER}{\ge}$$

$$\ge 6R\cdot\sqrt[3]{\frac{1}{4R\cdot\frac{R^2}{4}}}=6R\cdot\frac{1}{R}=6$$

Equality holds for $a=b=c$.