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UP.630. *Proposed by Daniel Sitaru, Romania.* Find:

$$\iint_D x^2 dx dy$$

where the region D is defined as: $D = \{(x, y) \mid 0 \leq y \leq 9 - x^2, -3 \leq x \leq 3\}$.

Solution by José Luis Díaz-Barrero, Barcelona, Spain. To evaluate the double integral, we set it up as an iterated integral

$$\int_{-3}^3 \int_0^{9-x^2} x^2 dy dx$$

First, integrating with respect to y , we get

$$\int_0^{9-x^2} x^2 dy = x^2 \left[y \right]_0^{9-x^2} = x^2(9 - x^2) = 9x^2 - x^4$$

Now, we integrate the resulting expression with respect to x , and we obtain

$$\int_{-3}^3 (9x^2 - x^4) dx = \left[\frac{9x^3}{3} - \frac{x^5}{5} \right]_{-3}^3 = \left[3x^3 - \frac{x^5}{5} \right]_{-3}^3 = \frac{324}{5}$$